Another Reason to Use the Hamilton Filter

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Abstract

This paper evaluates the sensitivity of filters commonly used in macroeconomics to data revisions. Using real-time data, I demonstrate that the Hamilton filter is significantly less sensitive to data revisions than the Hodrick Prescott filter and the Band-Pass filter; therefore, its application would allow real-time policy recommendations consistent from those obtained with the ex post revised data.

Keywords: Data Revisions, Filters, Real-Time Data.

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1 Introduction

Policymakers often base their policy decisions according to the information available to them in real time. A key variable for those decisions is the output gap, which can be approximated with the cyclical component of real GDP. However, the real-time cyclical component can differ considerably from the cyclical component obtained with the *ex post* revised data, which, as illustrated in Orphanides (2001), raises concerns about the "appropriateness" of policy recommendations made in real-time. To avoid this type of inconsistency, it would be optimal to use filters robust to data revisions.

In this note, I evaluate the sensitivity of three filters to data revisions: the Hodrick-Prescott (1981, 1997) filter, the Hamilton (2018) filter, and the Band-Pass filter of Christiano and Fitzgerald (2003). I apply each filter to real-time data and final vintages of a set of key macroeconomic series and compare the size of errors for estimated cyclical components across filters. I also compare a series of statistics such as correlation, standard deviation, autocorrelations, and cross-correlations. Finally, using Taylor's rule I evaluate the consistency between the real-time policy recommendation with those obtained with *ex post* revised data.

2 Filters and Data Revisions

Many macroeconomic variables are subject to data revisions, either because new data are released or because the information on which the series is built has improved. The objective of this paper is to analyze the sensitivity of different filters to data revisions. To be more precise, some concepts need to be defined.[1] Revisions are defined as any change in value for any reference point of the time series for a statistic released to the public. Revisions can occur when new observations become available, some current and/or past values are

[1]Definitions come from McKenzie and Gamba (2009).

modified, or there has been some methodological change. The vintage of a time series is defined as the set of data that represents the latest estimate for each reference point in the time series at a particular moment in time, and a real-time database is defined as a collection of historical vintages of the same time series, cataloged and indexed by the date on which each vintage became available to the public.

The Federal Reserve Bank of Philadelphia publishes the complete time-series history for each vintage for a set of key macroeconomic variables.[2] The series considered in this analysis correspond to GDP, personal consumption expenditures, investment, government expenditure, exports, and imports. The discussion is centered on the United States; however, to demonstrate that the results are not specific to this country, I also consider real-time data from Australia, Germany, New Zealand and the United Kingdom.[3] To assess the sensitivity of filters to data revisions, I use the following approach. The logarithm of each vintage is filtered. I build the "first release" series from the cyclical component at the end of the sample of each vintage. The last filtered vintage corresponds to the "final release" series.[4] To determine the sensitivity of the filter to data revisions, I use the following

[2] https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/real-time-data-set-full-time-series-history.

[3] Real-time series are published by the University of Melbourne Macroeconomics Research Unit for Australia, the Deutsche Bundesbank for Germany, the Reserve Bank of New Zealand for New Zealand, and the Bank of England for the United Kingdom.

[4] For United States, Australia, Germany and New Zealand, each vintage is published quarterly. For United Kingdom, each vintage is published monthly but I only consider the vintages published in the months of March, June, September and December, which correspond to the end of each quarter. The sample period of the vintages is 1970Q1-2020Q1 for the United States (start period 1947Q1 in general), 1980Q1-2017Q1 for Australia (start period 1959Q3 in general), 2005Q3-2020Q2 for Germany (start period 1991Q1), 2002Q3-2011Q3 for New Zealand (start period 1987Q2), and 1990Q1-2016Q3 for the United Kingdom (start period 1990Q1). For example, for United States the sample period for first release and final release series is 1969Q4-2019Q4 (1969Q4 is the end sample of 1970Q1 vintage and 2019Q4 is the end sample of 2020Q1 vintage). Note that, given the approach used, it is necessary to compare filters that do not lose observations at the end of the sample.

statistic:

$$S = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(cyclical_t^{\text{first release}} - cyclical_t^{\text{final release}}\right)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(cyclical_t^{\text{final release}}\right)^2}},$$

where $cyclical_t^{\text{first release}}$ and $cyclical_t^{\text{final release}}$ are the first release and final release cyclical component in period t, respectively.[5] This normalized value adjusts for differences that may exist in the volatility of the cyclical component across filters. A higher value of the statistic is indicative of a greater sensitivity of the filter to data revisions.

Table 1 shows the value of the statistic by filter type.[6] Note that, in general, the value of the statistic is lower for the Hamilton filter, followed by the Band-Pass filter. For all the countries considered, output, consumption, investment and imports report a lower statistic under the Hamilton filter. For exports, the Hamilton filter has a better performance in United States, Germany and United Kingdom, while, for government spending, the Band-Pass filter has a better performance in New Zealand and United Kingdom. For the few cases in which the Band-Pass filter presents a lower statistic than the Hamilton filter, the difference between the two statistics is small. Note that, for output and consumption, the statistics obtained from the Hamilton filter are considerably lower than the statistics of the other filters, especially in the United States.

Table 2 shows the correlation between the first release and final release series by filter type. For the United States, when the series are filtered with the Hamilton filter, the correlation between the first release and final release cyclical components is 0.95 for out-

[5]Figure 1-3 in the online Appendix 1 plot the first release and final release cyclical components for the real series of GDP, personal consumption expenditures, investment, government expenditure, exports and imports for United States by filter type.

[6]In the case of the United States, the investment made by the government is included within government spending, while, for the rest of the countries, the government investment is considered within the investment series.

put and 0.92 for consumption. On the other hand, when the series are filtered through the Band-Pass filter, the correlation is 0.77 for output and 0.78 for consumption, while, when the series are filtered through the HP filter, the correlation is only 0.53 for output and 0.51 for consumption. For the rest of the countries analyzed, the correlation between the first release and final release for the Hamilton filter is the highest for all series without exception. Using the correlation between the two series as a measure of similarity, the Hamilton filter produces very similar series from first and final releases.

Table 3 shows the standard deviation and the autocorrelations for the first release and final release cyclical components by filter type for the United States. It is important to highlight that, regardless of the series analyzed, the volatility of the cyclical component is greater when the series are filtered through the Hamilton filter.[7] The standard deviation of the final release series is quite similar between the HP filter and the Band-Pass filter. For example, the standard deviation of the output cyclical component is 1.46 for the HP filter and 1.43 for the Band-Pass filter, while, for the Hamilton filter, it is 3.10. When the series are filtered through the Hamilton filter or the HP filter, there are no significant differences (at least in percentage terms) between the standard deviation of the first release and final release cyclical component. On the other hand, when the series is filtered through Band-Pass filter, the standard deviation of the first release cyclical component is generally lower.

For output, consumption, investment and exports, the autocorrelation "match" between first release and final release is strikingly good when the series are filtered through the Hamilton filter. For instance, for consumption series, the mean square distance of the first

[7]In online Appendix 2, I show that the Hamilton filter is robust to the seasonal adjustment method. Table 2.1 shows the standard deviation of the cyclical component for 32 macroeconomic series extracted by applying the Hamilton filter, the HP filter, and the Band-Pass filter. A very interesting fact is that for most of the series analyzed, the standard deviation of the cyclical component is considerably greater when the series is filtered through the Hamilton filter.

five autocorrelations between releases is 0.007 for the Hamilton filter, 0.028 for the HP filter, and 0.047 for the Band-Pass filter. Also note that, regardless of the series analyzed, the autocorrelations decay more smoothly when the series are filtered through the Hamilton filter. Table 4 shows the cross-correlations between the variables analyzed for first release and final release cyclical components. The mean square distance of correlations between first release and final release matrices is 0.014 for the Hamilton filter, 0.023 for the HP filter, and 0.021 for the Band-Pass filter. Note also that for the HP filter, cross-correlations involving government spending change the sign between first release and last release.

Below I explore in more detail the main differences between first-release and final-release cyclical components for the output series.[8] The first striking fact is that, for both the HP filter and the Band-Pass filter, the greatest differences between both cyclical components tend to be observed in contiguous periods. For example, for the Band-Pass filter, the periods 1973Q2-1974Q1 and 2007Q4-2008Q2 represent periods of great differences between the cyclical component of the first release and final release. The period 1979Q2-1980Q2 represents a similar issue for the HP filter. In all these cases, the first release cyclical component is less than the final release cyclical component. In particular, for the period 1979Q2-1980Q2, the cyclical component of the first release HP filter is negative while the cyclical component of the final release is positive and greater than 2%. The same was observed for the Band-Pass filter in the period 2007Q4-2008Q2. The cyclical component of the first release is negative while the cyclical component of the final release is positive and greater than 1.5%. The previous results are of special interest since both periods are associated with an economic recession.

For the Hamilton filter, on the other hand, the greatest differences across releases occur

[8] In the online Appendix, table 1.1 reports a time series for first release and final release cyclical components of the output by filter type. For each type of filter, I highlighted the 10 periods in red where the greatest difference occurs (in absolute value) between the first release cyclical component and the final release cyclical component.

in isolated periods with the exception of 1995Q1-1995Q3. However, for that period, both for first release and final release, the filter concludes that the economy is above the trend. Also, it is not a period that stands out for abnormal growth rates. This may reflect a better adaptation of the Hamilton filter to turning points in the output series.[9]

A second striking fact is that, of the 201 periods analyzed, when the series are filtered through the Hamilton filter, in 188 periods, the sign of the cyclical component of the first release coincides with the sign of the cyclical component of the final release. In the case of the Band-Pass filter, the sign coincides in 148 periods, while, for the HP filter, the sign coincides in only 124 periods. In other words, when the series are filtered in real-time through the Hamilton filter and the cyclical component is obtained, in approximately 93.5% of the cases, the evaluation of whether the economic cycle is above or below the trend will coincide with the same evaluation when filtering the last available series. In contrast, when using the HP filter, matches only occur in 61.7% of the cases. Furthermore, the sign of the Hamilton cyclical component of the first release coincides with the sign of the HP cyclical component of the final release in 72.6% of the cases.

The previous point becomes vitally important if we take into account that policymakers often base their policy decisions according to the information available to them in real time.[10] Using Taylor's rule as an example, Orphanides (2001) demonstrates that real-time policy recommendations differ considerably from those obtained with *ex post* revised data. If the cyclical component of the series is obtained through the Hamilton filter, there will not be a significant difference in qualitative and quantitative terms between the decision that is made in real-time and the decision that would be made when observing the series

[9]Figure 4 in the online Appendix 1 plots by filter type, the difference between first release and final release cyclical component for output series, normalized by the standard deviation of the final release cyclical component.

[10]For example, the design of "automatic stabilizers" rely heavily on updated data (see Blanchard and Summers 2020).

ex-post. Recall that in the case of output, when the series are filtered through the Hamilton filter, the correlation between first release and final release series is 0.95.

To make the above point more apparent, suppose that the Federal Reserve makes its monetary policy decision based on the following Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_x x_t,$$

where π_t is inflation in period t and x_t is output gap in period t proxied with the cyclical component of output in period t.[11]

Figure 1 plots the error for each filter, defined as the difference between the real-time interest rate (output gap is proxied by the first release cyclical component) and *ex-post* interest rate (when the output gap is proxied by the final release cyclical component), normalized by the standard deviation of the final release cyclical component for output series. The vertical lines denote the NBER-dated onset of a recession. The size of errors is considerably larger when the series are filtered through the Band-Pass filter or HP filter (see bottom panel of Figure 1). Furthermore, the Band-Pass filter and the HP filter make larger errors on the dates when a recession starts while the Hamilton filter makes negligible errors on those dates, indicating that it is less sensitive to turning points. Consequently, the Hamilton filter is more likely to both detect a recession in real-time and make a consistent policy recommendation: if the Federal Reserve bases its monetary policy decision on the cyclical component obtained through the Hamilton filter in real-time, it can be relatively sure that, in the future, it will not regret its decision.

In simple words, the Hamilton cyclical component is interpreted as the difference between the value two years in the future and the value that we would have expected to see

[11] Taylor (1993) shows that $\phi_\pi = 1.5$ and $\phi_x = 0.5$ correspond to a good representation of the behavior of the Federal Reserve. I use these values for simulation.

based on its behavior until today. Note that this interpretation stands out for its simplicity. Since 1992, the Livingston Survey reports the annual-average real GDP forecast for the year in which the survey is conducted, and the annual-average real GDP forecast two years ahead of a group of professional forecasters. With this information, it is possible to build the expected growth rate of the economy for the next two years. I calculate the "actual" cyclical component as the difference between the actual growth rate and the expected two-year growth rate. The correlation between the actual cyclical component and the Hamilton cyclical component series is 0.88. On the other hand, the correlation between the actual cyclical component and the HP cyclical component is only 0.24, while the correlation between the actual cyclical component and the Band-Pass cyclical component is 0.43. That is, the cyclical component of Hamilton coincides with the deviations of output from the values predicted by specialists in economics.

3 Conclusion

The Hamilton filter is less sensitive to data revisions and its application generates policy decisions that are consistent over time. Given the simplicity of the Hamilton filter, its easy interpretation, and its good performance on the tests in this study, the Hamilton filter can become a standard filter to decompose a macroeconomic series into a cycle and trend.

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Table 1: Statistic Size of Errors by Filter

Filter	Output	Consumption	Investment	Government	Exports	Imports
United States						
Hamilton	0.36	0.39	0.38	1.26	0.59	0.45
HP						
$\lambda = 1600$	1.03	1.03	0.97	1.63	1.11	0.82
$\lambda = 400$	1.19	1.12	1.15	1.69	1.19	0.90
$\lambda = 100$	1.28	1.22	1.29	1.60	1.17	0.98
$\lambda = 10$	1.16	1.11	1.17	1.23	1.09	1.04
$\lambda = 1$	0.99	0.83	1.02	0.95	0.96	1.02
Band-Pass	0.67	0.65	0.66	1.14	0.87	0.62
Australia						
Hamilton	0.62	0.89	0.47	1.18	0.74	0.57
HP ($\lambda = 1600$)	1.02	1.02	0.90	1.29	0.97	0.78
Band-Pass	0.76	0.89	0.73	1.01	0.68	0.63
Germany						
Hamilton	0.35	0.63	0.66	0.83	0.46	0.35
HP ($\lambda = 1600$)	0.65	0.81	0.66	1.12	0.76	0.69
Band-Pass	0.63	1.04	0.71	0.81	0.58	0.55
New Zealand						
Hamilton	0.97	0.67	0.50	0.70	0.95	0.50
HP($\lambda = 1600$)	1.46	1.45	1.29	1.41	0.89	0.90
Band-Pass	1.01	1.18	1.00	1.35	0.89	0.74
United Kingdom						
Hamilton	0.62	0.55	0.66	0.84	0.74	0.70
HP($\lambda = 1600$)	1.23	1.54	1.14	1.43	0.94	1.13
Band-Pass	0.77	0.85	0.82	1.14	0.74	0.76

Source: Federal Reserve Bank of Philadelphia, University of Melbourne Macroeconomics Research, Deutsche Bundesbank, Reserve Bank of New Zealand and Bank of England.

Table 2: Correlation between First Release and Final Release Series

Filter	Output	Consumption	Investment	Government	Exports	Imports
United States						
Hamilton	0.95	0.92	0.93	0.55	0.94	0.90
HP						
$\lambda = 1600$	0.53	0.51	0.54	0.20	0.48	0.65
$\lambda = 400$	0.39	0.43	0.38	0.17	0.40	0.58
$\lambda = 100$	0.29	0.33	0.27	0.21	0.38	0.51
$\lambda = 10$	0.28	0.41	0.32	0.36	0.37	0.40
$\lambda = 1$	0.32	0.59	0.35	0.47	0.40	0.32
Band-Pass	0.77	0.78	0.76	0.67	0.57	0.81
Australia						
Hamilton	0.83	0.59	0.89	0.54	0.76	0.85
HP($\lambda = 1600$)	0.50	0.36	0.64	0.31	0.59	0.68
Band-Pass	0.72	0.52	0.70	0.47	0.74	0.79
Germany						
Hamilton	0.96	0.87	0.84	0.77	0.92	0.94
HP($\lambda = 1600$)	0.76	0.65	0.77	0.37	0.72	0.75
Band-Pass	0.89	0.22	0.78	0.64	0.87	0.88
New Zealand						
Hamilton	0.88	0.92	0.92	0.81	0.84	0.95
HP($\lambda = 1600$)	0.51	0.34	0.45	0.15	0.80	0.63
Band-Pass	0.61	0.55	0.55	0.24	0.75	0.67
United Kingdom						
Hamilton	0.87	0.87	0.78	0.67	0.82	0.86
HP($\lambda = 1600$)	0.46	0.30	0.42	0.02	0.59	0.52
Band-Pass	0.70	0.70	0.61	0.33	0.72	0.69

Source: Federal Reserve Bank of Philadelphia, University of Melbourne Macroeconomics Research, Deutsche Bundesbank, Reserve Bank of New Zealand for New Zealand and Bank of England.

Table 3: Standard Deviation and Autocorrelations of the Cyclical Component by Filter

Order	Hamilton		HP(λ = 1600)		Band-Pass	
	First Release	Final Release	First Release	Final Release	First Release	Final Release
Output						
SD	3.213	3.106	1.645	1.467	0.973	1.430
1	0.911	0.913	0.911	0.876	0.930	0.934
2	0.782	0.799	0.743	0.702	0.762	0.755
3	0.632	0.669	0.543	0.497	0.551	0.506
4	0.440	0.508	0.337	0.290	0.337	0.241
5	0.230	0.328	0.125	0.091	0.133	0.000
Consumption						
SD	2.747	2.797	1.289	1.192	0.796	1.178
1	0.928	0.929	0.874	0.888	0.942	0.944
2	0.826	0.827	0.745	0.743	0.807	0.790
3	0.701	0.713	0.602	0.569	0.638	0.568
4	0.541	0.557	0.445	0.351	0.450	0.316
5	0.364	0.395	0.239	0.143	0.249	0.067
Investment						
SD	11.439	11.334	5.565	5.510	3.620	5.581
1	0.933	0.932	0.926	0.925	0.953	0.948
2	0.839	0.828	0.800	0.777	0.836	0.804
3	0.719	0.700	0.640	0.593	0.676	0.598
4	0.565	0.543	0.471	0.397	0.498	0.367
5	0.390	0.365	0.283	0.199	0.313	0.139
Government						
SD	3.282	3.880	1.817	1.200	1.368	1.089
1	0.755	0.851	0.842	0.813	0.947	0.946
2	0.619	0.766	0.685	0.663	0.840	0.802
3	0.622	0.753	0.600	0.567	0.719	0.607
4	0.576	0.703	0.530	0.452	0.593	0.399
5	0.495	0.629	0.375	0.300	0.457	0.198
Exports						
SD	9.326	8.952	4.570	4.045	2.934	3.400
1	0.903	0.898	0.854	0.823	0.925	0.907
2	0.785	0.787	0.705	0.655	0.761	0.663
3	0.643	0.659	0.520	0.434	0.573	0.350
4	0.494	0.513	0.361	0.244	0.410	0.054
5	0.342	0.356	0.206	0.068	0.276	-0.176
Imports						
SD	10.030	10.230	4.701	4.858	2.918	4.692
1	0.930	0.922	0.817	0.830	0.903	0.921
2	0.831	0.811	0.638	0.622	0.688	0.713
3	0.710	0.684	0.447	0.420	0.456	0.446
4	0.572	0.543	0.277	0.228	0.270	0.192
5	0.426	0.386	0.121	0.070	0.149	-0.010

Source: Federal Reserve Bank of Philadelphia.

Table 4: Correlations by Filter

	First Release						Final Release					
Filter	Y	C	I	G	X	M	Y	C	I	G	X	M
Hamilton												
Y	1.00	0.90	0.87	0.18	0.40	0.80	1.00	0.88	0.87	0.17	0.36	0.85
C	0.90	1.00	0.81	0.14	0.14	0.77	0.88	1.00	0.83	0.10	0.09	0.85
I	0.87	0.81	1.00	-0.06	0.37	0.89	0.87	0.83	1.00	-0.14	0.19	0.86
G	0.18	0.14	-0.06	1.00	-0.11	0.08	0.17	0.10	-0.14	1.00	-0.24	0.06
X	0.40	0.14	0.37	-0.11	1.00	0.35	0.36	0.09	0.19	-0.24	1.00	0.23
M	0.80	0.77	0.89	0.08	0.35	1.00	0.85	0.85	0.86	0.06	0.23	1.00
HP($\lambda = 1600$)												
Y	1.00	0.89	0.88	0.22	0.42	0.79	1.00	0.87	0.90	-0.13	0.50	0.81
C	0.89	1.00	0.80	0.20	0.13	0.74	0.87	1.00	0.86	-0.10	0.20	0.78
I	0.88	0.80	1.00	-0.09	0.37	0.86	0.90	0.86	1.00	-0.35	0.31	0.85
G	0.22	0.20	-0.09	1.00	-0.16	0.01	-0.13	-0.10	-0.35	1.00	-0.19	-0.19
X	0.42	0.13	0.37	-0.16	1.00	0.39	0.50	0.20	0.31	-0.19	1.00	0.34
M	0.79	0.74	0.86	0.01	0.39	1.00	0.81	0.78	0.85	-0.19	0.34	1.00
Band-Pass												
Y	1.00	0.88	0.89	-0.03	0.43	0.81	1.00	0.89	0.92	-0.21	0.46	0.87
C	0.88	1.00	0.84	-0.09	0.13	0.77	0.89	1.00	0.91	-0.33	0.18	0.84
I	0.89	0.84	1.00	-0.32	0.40	0.87	0.92	0.91	1.00	-0.45	0.25	0.90
G	-0.03	-0.09	-0.32	1.00	-0.36	-0.14	-0.21	-0.33	-0.45	1.00	-0.08	-0.40
X	0.43	0.13	0.40	-0.36	1.00	0.34	0.46	0.18	0.25	-0.08	1.00	0.37
M	0.81	0.77	0.87	-0.14	0.34	1.00	0.87	0.84	0.90	-0.40	0.37	1.00

Source: Federal Reserve Bank of Philadelphia. Y: Output, C: Consumption, I:Investment, G: Government, X: Exports and M:Imports.

Figure 1: Error Interest Rate Series by Filter Type