Clientelism and Political Party Representation: A Deviation from the Median Voter Theorem*

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Preliminary

Abstract

We model a primary election within a political party in the presence of clientelism. The party membership consists of two groups: an ideological group and a clientelistic group. In this model, we show that political platforms diverge from the median as the proportion of clientelistic voters within the party increases. The classic Downsian result of convergence to the median is restored when this proportion tends towards zero.

The model also predicts that a candidate becomes more willing to purchase clientelistic votes from a political broker as the difference between the candidate's preferences and the party's median preference grows. Finally, in general elections where winning candidates from internal elections compete outside their party, the divergence caused by clientelism is significantly moderated.

Keywords: Clientelism, Auctions, Voting Behavior, Political Parties, Nash Equilibrium.

JEL: D72, D44, D02, C70.

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1 Introduction

The correct functioning of political parties is essential for a democracy. Parties represent political ideas and relevant sectors of society, articulating their demands to access power eventually. In this paper, we study whether the representation role of the political parties is being affected by political clientelism.[1]

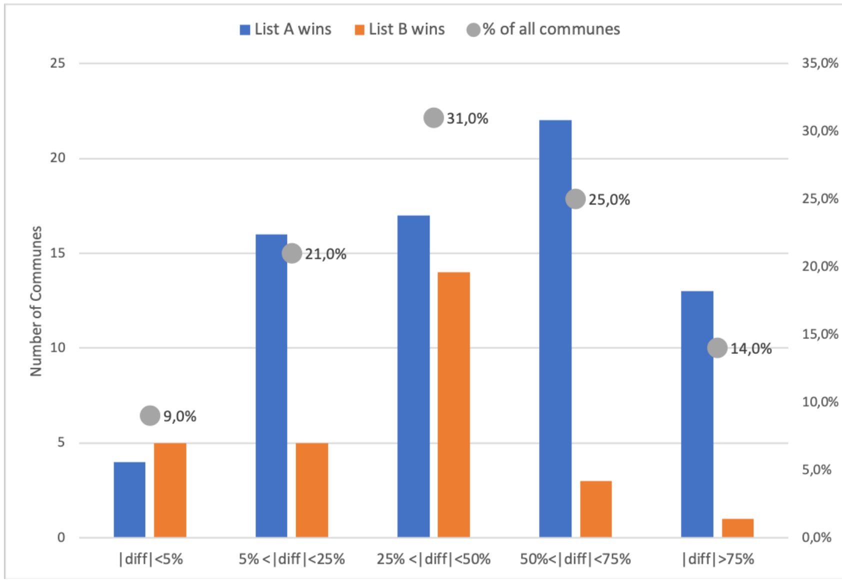
To illustrate this phenomenon, Figure 1 shows the results of the internal elections of the Chilean Socialist Party held out on May 24, 2019, which suggest the presence of clientelistic votes due to the high differences in votes received between lists per commune, which indicates that voters support a specific list en masse.[2] We take all the communes with at least 100 votes and see how the margin of victory of the winning list behaved. The first thing that stands out is that the margins by which the winning list won are pretty broad (an average of 31% over the losing list). Furthermore, in 39% of the communes, the winning list obtained a margin of more than 50% of the votes, while in 14% of the communes, the winning list managed to win by more than 75%.[3] Kitschelt and Kselman (2013) show that clientelism is quite widespread, even in Western European countries, but with greater prominence in countries of the former Soviet bloc and Latin America.

[1] Robinson and Verdier (2013) define clientelism as an exchange relationship between a patron and a "client." The patron provides some economic benefit in exchange for political support. As emphasized by Weingrod (1968), this economic benefit can take various forms, including public positions or favors, not limited to mere financial transfers. Desposato (2006) defines a clientelistic system as one where politicians provide some form of benefit before the election in exchange for votes that cannot be verified.

[2] Naturally, this fact does not necessarily imply clientelistic relations since these voting margins could be due to the political positions of some candidates being very far from the preferences of the militancy, which is not consistent with a Downsian competition model since, in that case, candidates should have proposals closer to the median, improving their chances of being elected. Another alternative explanation has to do with the fact that the median preferences differ considerably from one commune to another. In this way, a political proposal that is representative at the national level could not be representative at the commune level, which would also generate high voting margins.

[3] Francisco Vidal, Vice President of the Party for Democracy (PPD), denouncing clientelistic networks in Chile, said in a television program: "We lack a brutal internal democracy. The last time I applied for the political commission, I got about 400 votes; I called by phone, but oh, surprise! When the night of the counts, I saw the commune of Colina; we were 104 candidates, Vidal 93 votes, all the rest zero, and I never called Colina. Is it reasonable that someone receives 100 percent of the votes in a commune of militants? No. La Pintana Commune is a nice commune with two hundred people voting actively. Samuel Donoso took all the votes, whom no one knew in La Pintana."

Figure 1: Percentage of Communes by Difference in Votes



Notes: The figure shows the results of the internal elections of the Chilean Socialist Party held on May 24, 2019. The gray points show the percentage of communes by difference in votes between list A and list B (in absolute value). The blue bar shows the number of communes in which List A won, while the orange bar indicates the number of communes in which List B won. We consider communes with at least 100 votes.

Motivated by this evidence, we build a model representing an internal election within a political party in which ideological and clientelist militants coexist. The "clients" provide political support to a candidate, regardless of their political platform, in exchange for economic compensation. We assume the presence of a broker or "political broker" who directs their clients to vote for a particular candidate. In this way, we can infer what incentives generate clientele networks within a political party and how these distort its internal democracy and thus alter its ability to represent militancy. We prove the existence and characterize Nash equilibria and see how they depend on the relative importance of clientelist militants.

From this formalization, we infer a series of exciting results. A higher level of clientelism within a political party accentuates the divergence of the political platforms to the median of the political party's preferences, reducing the militancy's political representation. We also show that candidates with ideological preferences furthest from the median of the party are more willing to use clientelist networks. Furthermore, the abstention of non-clientele militancy reinforces this divergence in political platforms, so low electoral participation could be convenient for those candidates who use clientelist networks. Finally, we show that vote-buying diverges the platform of those who buy votes and allows candidates who do not use clientelistic networks to move their platforms away from the median since non-clientele militancy has a lower relative weight over the total votes.

Next, we analyze the case in which there are general elections in which the winning candidates of the internal elections of the political parties (primary elections) compete. We show that the existence of general elections moderates the divergence of chosen political platforms from the median of the political party's preferences relative to the case without general elections, reducing the value that the candidates give to the purchase of votes. This result implies that in political systems with low competitiveness between parties, the value of using clientelistic networks will generally be more significant than in political systems with high electoral competition between parties.

Finally, it's important to note that, up to this point, we have assumed that each candidate maximizes the number of votes received in the election. However, this assumption is not realistic in all cases. In a simple poll like the one we have described, it is more reasonable to assume that the candidate is maximizing the probability of being elected. We recognize that, in general, maximizing the likelihood of winning an election does not yield the same outcome as maximizing the quantity or proportion of votes received. Given this, we reformulate the baseline model, focusing on maximizing the probability of winning. We introduced a random component into voters' preferences to derive an expression for the likelihood of victory. As a result of this exercise, we find that the conclusions of the primary model remain intact: Vote-buying increases the divergence of platforms from the median, and the willingness to buy votes increases with the proportion of votes controlled by the broker.

Related literature. In an influential article, Downs (1957) shows that candidates tend to offer the same political platform, coinciding with the median preference of the votes. This result is known as the Median Voter Theorem. This insight had been previously anticipated by Hotelling (1929) in the context of two companies choosing their positions within a continuum of potential buyers, revealing that they eventually position themselves at the median of customer preferences. This remarkable finding has gained further support through the inclusion of political preferences not only among voters but also among candidates (see Austen-Smith and Banks (2009) and Banks and Duggan (2005)). Notably, pure strategies are not generally assured unless randomness is introduced into certain parameters, such as by adding preference shocks. In this vein, the present work demonstrates how clientelism within a party weakens the Downsian result, illustrating how the possibility of employing client networks enables candidates to win elections without needing to align their proposals with the party's median.

Another strand of literature addresses distortions of the median resulting from unequal campaign spending among candidates. Austen-Smith (1987) argues that candidates face a trade-off between moving away from the median to secure campaign resources versus approaching the median to win votes. Baron (1994) presents a similar trade-off but considers the possibility of informed voters immune to campaign spending alongside uninformed voters who can only be reached through such expenditures. In this context, the level of resources a candidate demands will be higher as the proportion of uninformed voters increases. Along these lines, Grossman and Helpman (1996) show that candidates behave as if they are maximizing the weighted sum of preferences from interest groups and informed voters. In a different approach, Prat (2003) demonstrates that lobbying serves as a signaling mechanism in the presence of information asymmetry, where businesses tend to support candidates with better abilities in equilibrium. Thus, candidates who spend more due to higher contributions signal their greater capacity to perform in office. In contrast to exploring how candidates move away from the median to secure campaign funds, this study reveals how using client networks allows candidates to give less weight to the preferences of the party's membership.

This paper is also closely related to the literature on runoff elections. Riker (1982) Duverger's Hypothesis posits that this system encourages multipartyism. Haan and Volkerink (1996) restore the Downsian result in cases where a single-round election doesn't preserve it. Similarly, Bordignon and Tabellini (2016) shows that runoff elections moderate political extremism in proposed platforms. Lastly, Brusco, Dziubinskia, and Roy (2012) illustrate that in the presence of runoff elections, multiple equilibria generally exist, with convergence to the median being just one possible outcome. This study aligns with existing literature, showing that the effect of clientelism is moderated when victorious primary candidates must subsequently compete in a general election, which plays a comparable role, in this context, to a runoff election.

Finally, this paper falls within the literature of Political Science that deals with informal political institutions. These refer to socially shared, unwritten, sanctioned norms and practices outside official channels. These informal relations encompass clientelism, patronage, and corruption among government officials. One potential effect of these institutions is the limitation of the representational capacity of formal political institutions. Desposato (2006) illustrates that a greater prevalence of clientelism within a political party diminishes its ability to represent voters programmatically. Levitsky and Helmke (2006) delve into the nature of clientelistic relations and, in broader terms, the informal institutions within political party systems. Kitschelt (2011), using cross-country data, shows that clientelism transcends political regimes, with socio-economic variables of the population holding more significance. Robinson and Verdier (2013) demonstrate that if clientelistic relationships focus on providing jobs, the efficiency in delivering public goods lowers. Empirical work by Stokes (2005) and Finan and Schechter (2012) underscores the importance of monitoring and reciprocity in maintaining clientelistic relationships.

This paper is organized as follows. Section 2 develops an election model within a political party in the presence of clientelist voters. Section 3 characterizes the existence of Nash equilibria. In section 4, we show the model's main predictions about the effects of clientelism on political parties' political platforms. Next, we incorporate two extensions to the baseline model. In section 5.1, we analyze the case where the winners of internal elections of the political parties compete in a general election, while in section 5.2, we maximize the probability of winning instead of the number of votes. Section 6 concludes.

2 Election Model

2.1 Model Agents: Party Members, Candidates, and Brokers

We present the election model within a political party. The "clients" provide political support to a candidate, regardless of their political platform, in exchange for economic compensation.[4] We assume the presence of a broker or "political broker" who directs their clients to vote for a particular candidate.

Because parties are not entirely clientelistic but also have a patronage component and an ideological one, we assume the existence of a share of party members who vote based on their ideology: They vote for the candidate proposing the platform closest to their ideal political stance or "bliss point." These voters behave according to classical Downsian competition.

We next describe in detail the participants in this election and the game's rules for the election and vote-buying. Proofs of all propositions are relegated to Appendix A.3.

1. **Party Members:**

   Party members are divided into two groups. Firstly, an "ideological" group of party members is characterized by each member voting for the candidate proposing the platform closest to their preferred or "bliss point" platform. These preferences are uniformly distributed[5] in the interval $[-1/2, 1/2]$, meaning $F(\cdot) \sim$

[4]For this model, we are simplifying clientelism to the vote-buying managed by a broker. Clientelism is a much more complex phenomenon, where the relationship between clients and brokers is based on long-term relationships, where clients can access public jobs or directly obtain goods from the government or the politicians themselves. We don't model these features; instead, we assume the existence of a political broker who directs their clients to vote for one of the candidates.

[5]The results are not dependent on using a uniform distribution. In Appendix A.1, we show that the results are robust to the distribution used.

$\mathcal{U}[-1/2, 1/2]$.[6] This ideological group represents a share $(1-\alpha)$ of all party members.

Secondly, there are non-ideological or clientelist party members. They won't vote according to the platforms proposed by the candidates; instead, they vote as a bloc for the candidate favored by the broker. This group represents a share α of all party members.

2. **Candidates:**

Two candidates are indexed by $i \in \{1, 2\}$. Each candidate's preferences have three components: (i) candidates want to win the election, (ii) they have ideological preferences and penalize deviations from their preferred ideology (bliss point), and (iii) monetary resources can be used to buy the support of political brokers.

Specifically, each candidate's utility function is given by

$$\pi_i(\varphi_i, p_i, \beta_i) = \varphi_i - \delta_i \left(p_i - \overline{p_i}\right)^2 + m(R_i - \beta_i), \tag{1}$$

where φ_i corresponds to an electoral success function. For the baseline model, this function corresponds to the proportion of votes obtained from the total party membership for candidate i. p_i is the candidate's proposed political platform, while $\overline{p_i}$ is the candidate's "bliss point." δ_i measures the disutility of deviation from this point.

Lastly, β_i represents the payment made to the broker for a quantity of votes α_i. Each candidate has a limited amount of resources R_i, which we assume to be exogenous, to allocate towards buying votes from the broker. Hence, the condition

$$R_i - \beta_i \geq 0$$

[6] It's worth noting that in this model, we maximize the fraction of votes obtained, which is not equivalent, except in specific cases, to maximizing the probability of winning the election. In section 5.2, we develop a model where we maximize the probability of victory. Additionally, we note that this model could be grounded in a probabilistic voting model: Duggan (2005) shows that when each voter has some degree of randomness in their utility function, this randomness cancels out when aggregating all voters.

holds. Finally, m is a strictly positive parameter measuring the marginal utility of money.[7]

3. **Broker:**

   A broker manages a fraction α of the total party membership. The broker seeks to maximize their income β by selling the votes α under their control. We don't model how the broker maintains client networks and ensures clients vote for the favored candidate. Thus, we assume that all clientelist party members vote for the candidate supported by the broker.[8] It's important to highlight that we assume complete independence between the candidate's ideology and the broker's support. However, as Calvo and Murillo (2016) show this is not necessarily true, and certain types of parties may use more client networks than others. In this case, we assume that both candidates have the same availability to use client networks, as they both belong to the same party.

2.2 Election Characteristics

In this election, the ideological members experience aversion towards voting for candidates using the broker's clientelistic relationship: The votes obtained by each candidate suffer a penalty of $\lambda_i \alpha_i$ where λ_i captures the aversion to voting for a candidate using clientelistic votes. Therefore, the share of votes obtained by each candidate, conditional on the proposed platforms, is

$$V_1 = \alpha_1 + (1-\alpha)(1-\lambda_1\alpha_1)F(p_m) = \alpha_1 + (1-\alpha)(1-\lambda_1\alpha_1)\left(\frac{p_1+p_2}{2}+\frac{1}{2}\right) \tag{2}$$

[7]The parameter m will only affect the scale of the willingness to pay so that we normalize it to $m=1$. The budget constraint R_i can be understood more broadly as the maximum amount of money to be transferred, as it can also represent the candidate's capacity to provide other types of favors.

[8]Note that this model is similar to Grossman and Helpman (1996) in that both models are based on dividing the electorate between voters who vote based on candidates' proposals and those who do not. However, unlike Grossman and Helpman (1996) where the candidate aligns their stance with their financiers' in exchange for campaign money, here the candidate uses a sum of money to buy a fraction of the votes, thus aligning their platform closer to their own "bliss point." Naturally, the source of the candidate's resources could be modeled, arriving at a synthesis between the model presented in this work and that of Grossman and Helpman (1996).

$$V_2 = \alpha_2 + (1-\alpha)(1-\lambda_2\alpha_2)\left(1-F(p_m)\right) = \alpha_2 + (1-\alpha)(1-\lambda_2\alpha_2)\left(\frac{1}{2}-\frac{p_1+p_2}{2}\right), \quad (3)$$

where $p_m = (p_1+p_2)/2$ is the average proposed platform.[9] If no one buys the broker's votes, the share of votes is given by

$$V_1 = F(p_m) \quad (4)$$

$$V_2 = 1 - F(p_m). \quad (5)$$

Notice that the candidate closest to the median would win the election in this latter case. Thus, it reverts to the classical Downsian result.

2.3 Game Timing

The game consists of two stages. In the first stage, each candidate decides whether to participate in a second-price auction in which the broker will deliver their votes to the candidate making the highest bid.[10] If they win the auction, candidate "i" receives all the available votes. In other words, the broker cannot sell a portion of votes to one candidate and the remaining to the other. Suppose no one participates in the auction.[11] In that case, the broker's votes will not be added to any candidate.

In the second stage, the candidates compete in the party's internal election or primary. They choose their political platforms p_i optimally and make payments β_i. The set of feasible strategies of candidate "i" is

$$(p_i, \beta_i) \in \left([-1/2, 1/2], \{\phi\} \cup \mathbb{R}_+\right).$$

To account for the possibility of a candidate not participating, we set $\beta_i = \phi$. Finally,

[9] Here, we assume that $1 - \lambda_i\alpha_i \geq 0$ for all i, giving us an upper bound for the penalty for buying votes: $\lambda_i \leq \frac{1}{\alpha_i}$.

[10] This assumption allows us to find the optimal bids of each candidate quickly. The optimal strategies do not depend on each candidate's valuation distribution, so we don't need to model it. It's different in the case of, for example, a first-price auction where the optimal bid depends on the distribution of each player's valuation. See Krishna (2009) for an exposition on equilibrium bidding for different types of auctions.

[11] In the next section, we show that this case can be relevant under certain parameter combinations.

the equilibrium concept will be that of subgame perfect Nash equilibrium: The chosen equilibrium must be an equilibrium in every proper subgame.[12]

3 Equilibrium

3.1 Resolution of the Second Stage

To find a perfect subgame equilibrium, we must solve the problem recursively, starting from the last stage. In this stage, $\{\alpha_i, \beta_i\}_{i=1,2}$ are treated as given. Then, each candidate maximizes

$$\max_{p_i} \pi_i = V_i - \delta_i (p_i - \overline{p_i})^2,$$

where V_i corresponds to the fraction of votes defined in equations (2) and (3).

Proposition 1. *If candidate 1 buys the votes, the proposed platforms for each candidate are*

$$\widehat{p_1}(\alpha, 0) = \overline{p_1} + \frac{(1-\alpha)(1-\lambda_1\alpha)}{4\delta_1} \tag{6}$$

$$\widehat{p_2}(\alpha, 0) = \overline{p_2} - \frac{(1-\alpha)}{4\delta_2}. \tag{7}$$

If candidate 2 buys the votes, the proposed platforms are

$$\widehat{p_1}(0, \alpha) = \overline{p_1} + \frac{(1-\alpha)}{4\delta_1} \tag{8}$$

$$\widehat{p_2}(0, \alpha) = \overline{p_2} - \frac{(1-\alpha)(1-\lambda_2\alpha)}{4\delta_2}. \tag{9}$$

[12] The model has some limitations. The broker delivers the votes regardless of the candidate's ideology, which is not necessarily a realistic assumption in all cases. In this sense, a possible deepening of the model would involve modeling the existence of more than one broker so that an ideological match occurs between the candidate and the broker. Another limitation of this model is that it takes as exogenous the resources each candidate has to buy the broker's votes. In this sense, it would be interesting to model obtaining these resources from financiers, like the hybrid model of Grossman and Helpman (1996).

Finally, if neither candidate buys the votes, the platforms are

$$\widehat{p_1}(0,0) = \overline{p_1} + \frac{1}{4\delta_1} \tag{10}$$

$$\widehat{p_2}(0,0) = \overline{p_2} - \frac{1}{4\delta_2}. \tag{11}$$

□

Proof. See Appendix A.3.1. ■

Note that equations (10) and (11) represent the political platforms closest to the party median. In that sense, they are considered competitive benchmarks. However, it's worth emphasizing that the median voter theorem (i.e., $\widehat{p_1} = \widehat{p_2} = 0$) doesn't hold, as candidates have ideological preferences that create an incentive to reduce their winning probability in exchange for moving closer to their bliss point. In section 4, we study the implications of these results.

3.2 Resolution of the First Stage

Prior to proposing the optimal platforms $\widehat{p_1}(\alpha_1, \alpha_2)$ and $\widehat{p_2}(\alpha_1, \alpha_2)$, each candidate will offer a transfer β_i to the local leader in order to obtain their votes α.

Proposition 2. *We define π_{ij} as the utility received by candidate i when candidate j buys the votes. In other words, $\pi_{11} = \pi_1(\widehat{p_1}(\alpha,0), \widehat{p_2}(\alpha,0))$, $\pi_{12} = \pi_1(\widehat{p_1}(0,\alpha), \widehat{p_2}(0,\alpha))$, $\pi_{22} = \pi_2(\widehat{p_1}(0,\alpha), \widehat{p_2}(0,\alpha))$, and $\pi_{21} = \pi_2(\widehat{p_1}(\alpha,0), \widehat{p_2}(\alpha,0))$, where $\widehat{p_i}(\alpha_1,\alpha_2)$ are the optimal platforms found in equations (6) and (7). Additionally, we define π_{10} and π_{20} as the utility candidates receive when no one buys the votes.*

Suppose that for all i, $\pi_{ii} > \pi_{ij}$ and $\pi_{i0} < \max\{\pi_{ij}, \pi_{ii}\}$. Then, the optimal offers made by the candidates to the political broker are

$$\beta_1^* = \min\{\Phi_1, R_1\} \quad \textit{and} \quad \beta_2^* = \min\{\Phi_2, R_2\}, \tag{12}$$

where

$$\Phi_1 \equiv (\pi_{11} - \pi_{12}) \quad \textit{and} \quad \Phi_2 \equiv (\pi_{22} - \pi_{21}) \,. \tag{13}$$

□

Proof. See Appendix A.3.2. ■

The willingness-to-pay Φ_i corresponds to the incremental benefit of winning the auction versus the scenario of losing it and letting the other candidate win.

The model parameters (λ_i, δ_i, $\overline{p_i}$, and α) determine whether the assumptions of Proposition 2 hold. This case corresponds to the "expected" scenario, as at least one candidate values buying votes positively, preferring that situation over the competitive *status quo*. However, with very high values of λ_i and very low values of δ_i, buying votes could harm the candidate. Furthermore, vote-buying could reduce the benefits for both the candidate buying them and their competitor. In subsection 3.3, we explore those cases and demonstrate that, in general, we cannot ensure the uniqueness of the equilibrium.

3.3 Existence and Uniqueness in the First Stage

The auction in the first stage corresponds to an auction with identity externalities (IDE). In a typical auction, participants are indifferent about who wins the auction if it's not themselves. In IDE auctions, on the other hand, losers receive utility (or disutility) based on who won the prize. A classic example given in Funk (1996) is the auction of an important painting, where a losing museum would prefer the artwork to go to a private collector rather than another museum, as the private collector might potentially attract fewer visitors from them. The critical point in such auctions is that the willingness to pay becomes endogenous to the equilibrium, so it can only be computed once the auction is concluded.

As highlighted by Jehiel and Moldovanu (1996), externalities create potential strategic interactions among players that could discourage participation or even incentivize a player who doesn't value the object to participate solely to prevent a specific player from winning the auction.[13]

Funk (1996) demonstrates the existence of Nash equilibria in pure strategies in first-price auctions with externalities but shows that equilibrium is not unique. Klose and Kovenock (2015) do the same for an all-pay auction with externalities, establishing the existence of multiple equilibria and characterizing them. To the best of our knowledge, a result of the existence of equilibrium in second-price auctions with externalities for the general case has not been established. Of course, this doesn't imply that equilibrium cannot exist in specific cases.

We present two cases in Proposition 3 depending on parameters. The first case is an equilibrium for no candidate to participate in the vote-buying auction. This equilibrium is not necessarily unique. For the second case, a specific case of the first, we show two equilibria in pure strategies and one in mixed strategies.

Proposition 3. *Consider the following cases:*

Case 1*: Suppose that*

$$\pi_{i0} > \pi_{ii} \ \ \forall i.$$

Then, the equilibrium outcome is that neither candidate participates in the auction to obtain votes, denoted by

$$\beta_1^* = \beta_2^* = \phi,$$

with payments (π_{10}, π_{20}).

[13] In the above example, a museum that doesn't value the painting could participate and try to obtain it solely to prevent its competitor from getting it. This example questions the suitability of auctions as mechanisms for maximizing social welfare in the context of externalities.

***Case 2**: Suppose that*

$$\pi_{12} < \pi_{11} < \pi_{10}$$
$$\pi_{21} < \pi_{22} < \pi_{20}.$$

Then, there are two equilibria in pure strategies and one equilibrium in mixed strategies:

- $\beta_1^* = \beta_2^* = \phi$ *with payments* (π_{10}, π_{20}).
- $\beta_1^* = \min\{\Phi_1', R_1\}$ *and* $\beta_2^* = \min\{\Phi_2', R_2\}$ *, where* $\Phi_1' \equiv \max\{0, \pi_{1,1} - \pi_{12}\}$ *and* $\Phi_2' \equiv \max\{0, \pi_{2,2} - \pi_{21}\}$. *The payments are* (π_{12}, π_{22}).
- *Equilibrium in mixed strategies where the participation probability is*

$$(\sigma_1^*, 1-\sigma_1^*) = \left(\frac{\mathbb{E}(\pi_{2\bullet}) - \pi_{21}}{\pi_{20} - \pi_{22} + \mathbb{E}(\pi_{2\bullet}) - \pi_{21}}, \frac{\pi_{20} - \pi_{22}}{\pi_{20} - \pi_{22} + \mathbb{E}(\pi_{2\bullet}) - \pi_{21}}\right)$$
$$(\sigma_2^*, 1-\sigma_2^*) = \left(\frac{\mathbb{E}(\pi_{1\bullet}) - \pi_{12}}{\pi_{10} - \pi_{11} + \mathbb{E}(\pi_{1\bullet}) - \pi_{12}}, \frac{\pi_{10} - \pi_{11}}{\pi_{10} - \pi_{11} + \mathbb{E}(\pi_{1\bullet}) - \pi_{12}}\right),$$

  with $0 < \sigma_i^* < 1$ *for all* i. *The offers of the candidates, in case of participation, are*

$$\beta_1^* = \min\{\Phi_1', R_1\} \text{ and } \beta_2^* = \min\{\Phi_2', R_2\},$$

  where $\Phi_1' \equiv \max\{0, \pi_{1,1} - \pi_{12}\}$ *and* $\Phi_2' \equiv \max\{0, \pi_{2,2} - \pi_{21}\}$.

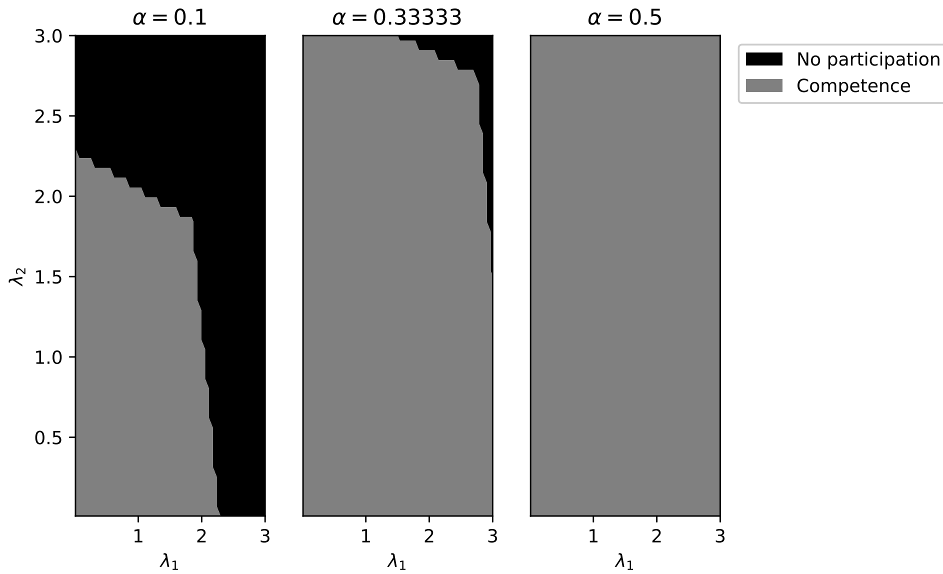
□

Proof. See Appendix A.3.3. ■

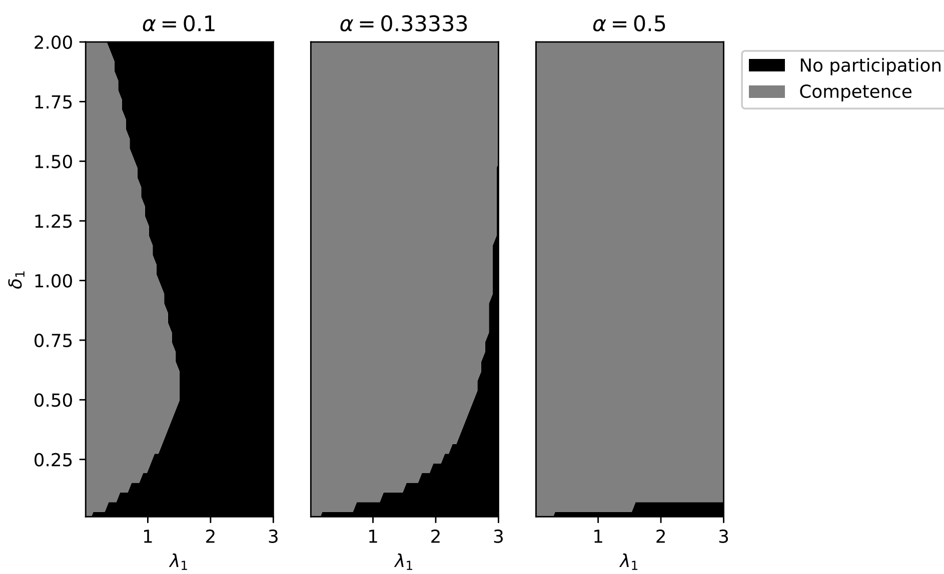
Figure 2 presents a range of parameter values that lead to different equilibria. The grey areas illustrate the combination of parameters where both candidates value the votes (**Proposition 2**), which implies participation in the auction. In contrast, black areas indicate a combination of parameter values where the assumptions of Proposition 2 are not met, leading to potential equilibria involving non-participation and mixed strategies (**Proposition 3**).

Figure 2: Equilibria under different combinations of parameters

(a) Interaction of λ_1 and λ_2



(b) Interaction of λ_1 and δ_1



Notes: The figure presents a range of parameter values that lead to different equilibria. Panel (a) sets $\overline{p_1} = -0.5$, $\overline{p_2} = 0.5$, $\delta_1 = \delta_2 = 1$ and interact λ_1 and λ_2, Panel (b) sets $\overline{p_1} = -0.5$,$\overline{p_2} = 0.5$, $\lambda_2 = 2$, $\delta_2 = 1$ and interact λ_1 and δ_1.

4 Main Results: Comparative Statics

4.1 Effect on Proposed Platforms and Median Divergence

Next, we draw results based on the previous propositions. Proofs of all results are relegated to Appendix A.3. Regarding the optimal platforms in the second stage (equations (6)-(11)), we have the following results:

Result 1 (Broker Effect). *The platform of the candidate who buys votes diverges more from the median than the competitive benchmark ($\widehat{p}_1(0,0)$ and $\widehat{p}_2(0,0)$).*

$$|\widehat{p}_1(\alpha,0) - 0| > |\widehat{p}_1(0,0) - 0|$$
$$|\widehat{p}_2(0,\alpha) - 0| > |\widehat{p}_2(0,0) - 0| .$$

□

Proof. See Appendix A.3.4. ■

This result shows that buying votes implies a more significant divergence from the median when the candidate buys the votes because the voting of the clientele segment of the party's membership reduces the weight of the ideological part of the party. Consequently, moving closer to the median of this group becomes less valuable. This divergence is intensified by the electoral abstention generated by the parameter λ, making the ideological membership even less relevant.

Result 2 (Complementary Corruption). *The candidate who doesn't buy votes diverges more from the competitive benchmark than if no one buys votes.*

$$|\widehat{p}_1(0,\alpha) - 0| > |\widehat{p}_1(0,0) - 0|$$
$$|\widehat{p}_2(\alpha,0) - 0| > |\widehat{p}_2(0,0) - 0| .$$

□

Proof. See Appendix A.3.4. ■

This effect is less intuitive. We see that the use of corrupt votes not only causes the political platform of the candidate who uses them to diverge but also the one who doesn't use them. This result is because ideological membership becomes less relevant regardless of who buys votes, leading to fewer incentives to cater to them. Naturally, for a candidate without ideological preferences ($\delta_i = 0$ or very low), this effect may not apply or would be less relevant, as the candidate's interest would solely lie in winning the election.[14]

Result 3 (Membership Effect). *Platforms diverge more from the median as the number of non-ideological members increases.*[15]

$$\begin{aligned}
\frac{d\widehat{p}_1(\alpha,0)}{d\alpha} &= \frac{1}{4\delta_1}\left(-1-\lambda_1+2\alpha\lambda_1\right) < 0 \\
\frac{d\widehat{p}_2(\alpha,0)}{d\alpha} &= \frac{1}{4\delta_2} > 0 \\
\frac{d\widehat{p}_1(0,\alpha)}{d\alpha} &= -\frac{1}{4\delta_1} < 0 \\
\frac{d\widehat{p}_2(0,\alpha)}{d\alpha} &= \frac{1}{4\delta_2}\left(1+\lambda_1-2\alpha\lambda_2\right) > 0
\end{aligned}$$

□

Proof. See Appendix A.3.4. ■

This effect tells us that as the ideological group of party membership becomes less influential, the incentives for candidates to move towards the median decrease. In fact, in the limiting case where the party is entirely clientelist (i.e., $\alpha \to 1$), the candidates' platforms tend toward their respective bliss points, $\overline{p_1}$ and $\overline{p_2}$.

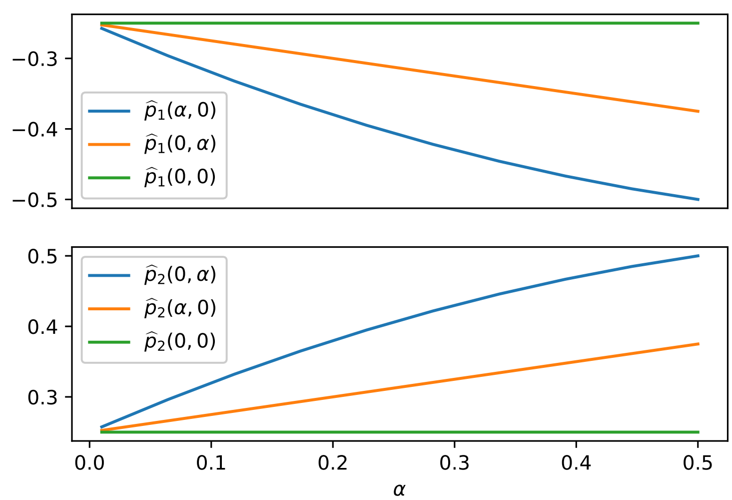
[14]This last intuition holds because the reward for getting closer to one's bliss point is independent of whether they win the election or not. In the hypothetical case of a candidate having zero chance of winning the election, they would be incentivized to propose exactly their bliss point.

[15]This holds for $\alpha < 1/2$.

These results are summarized in Figure 3. As we see, the scenario in which candidates must get closer to the party median is when no one buys votes (i.e., $\widehat{p_1}(0,0)$ and $\widehat{p_2}(0,0)$). On the other hand, Candidate 1 (or Candidate 2) moves further away from the median when their rival takes the votes than when no one buys them. However, the scenario in which they move furthest from the median is when they get the votes. Furthermore, it is worth noting that as α grows, the divergence from the median increases.

These three results seem to contradict Huntington (1968), who argues that political systems with clientelist or patronage-based parties tend to be less ideological. However, the results presented in this paper show the distortion in the representation of party preferences. In this context, if the candidate buying votes had "centrist" preferences, the effect would be that the winning platform becomes centrist. The critical outcome is the distortion in representing party members' preferences.

Figure 3: Optimal platforms



Notes: The figure plots the optimal platforms $\widehat{p_1}$ and $\widehat{p_2}$ for different values of α. We set $\overline{p_1} = -0.5$, $\overline{p_2} = 0.5$, $\lambda_1 = \lambda_2 = 1$, and $\delta_1 = \delta_2 = 2$. Budget constraints are not active.

4.2 Effect of Polarization on Vote-Buying

In this section, we focus on the vote-buying stage. We show that the further a candidate is from the party's median, the greater their willingness to pay for the broker's votes.

Result 4 (Extremism and Greater Willingness to Buy Votes). *Suppose that R_1 and R_2 are sufficiently large such that $\Phi_i = \beta_i$.*[16] *Then, it holds that*

$$\Phi_1 \geq \Phi_2 \iff |\overline{p_1}| \geq |\overline{p_2}|$$

□

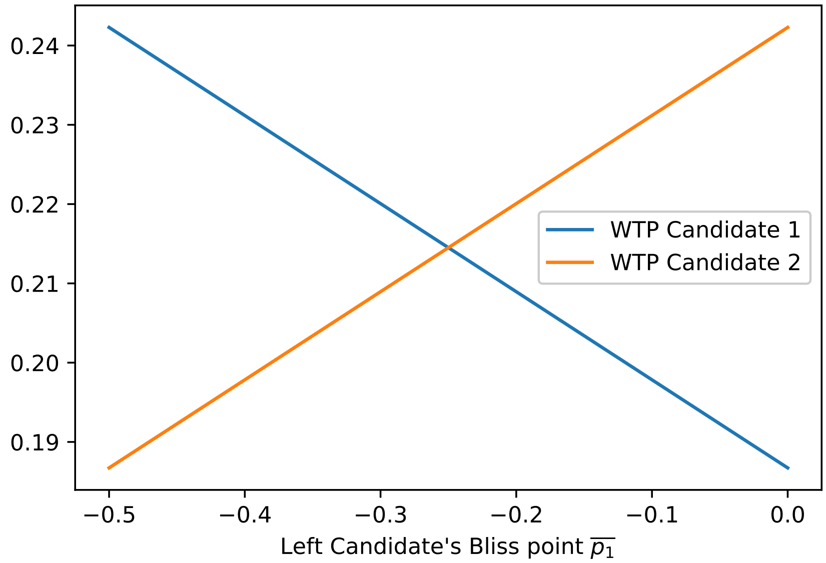
Proof. See Appendix A.3.5. ■

The preceding result indicates that a candidate who is farther from the party's median (i.e., $|\overline{p_i} - p_m| = |\overline{p_i}|$) has a greater willingness to buy the broker's votes. Moreover, we can demonstrate that the willingness to pay Φ_i increases with the distance of the candidate's bliss point from the median. As shown in Figure 4, when the bliss points are equal ($|\overline{p_1}| = |\overline{p_2}| = 0.25$) , the willingness to pay is also equal. However, when Candidate 1 is farther from the median ($|\overline{p_1}| > 0.25$), it is this candidate who is willing to pay more to buy votes. The opposite occurs when the more extreme candidate is Candidate 2. Also evident from Figure 4 is that as polarization increases, the willingness to pay also increases. At the same time, as preferences approach the median, the willingness to buy votes decreases.

Another intriguing effect is shown in Figure 5. As the disutility of deviation from the bliss point of Candidate 1 (δ_1) increases, the willingness to pay grows. Conversely, if the penalty for deviating from the bliss point decreases, the willingness to pay decreases.

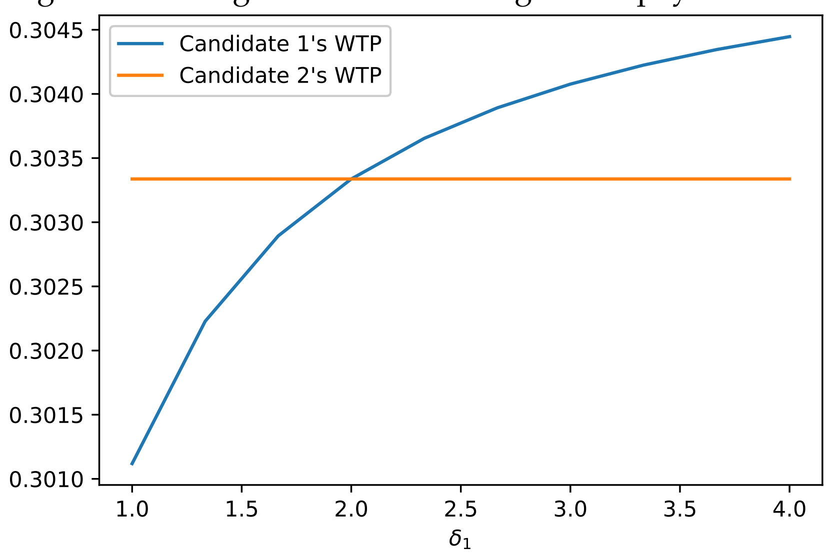
[16] A sufficiently large number $\overline{R}$ is the highest utility between both candidates, obtained by evaluating each benefit function at the maximum possible votes, at the bliss point, and without spending resources, i.e., $\overline{R} = \max_i \pi_i(1, \overline{p_i}, 0)$.

Figure 4: Extremism and willingness to pay for votes



Notes: The figure plots the willingness to pay for votes of both candidates depending on Candidate 1's bliss point. We set $\overline{p_2} = 0.25$, $\lambda_1 = \lambda_2 = 1$, $\delta_1 = \delta_2 = 2$, and $\alpha = 1/3$. Budget constraints are not active.

Figure 5: Ideological cost and willingness to pay for votes



Notes: The figure plots the willingness to pay for votes of both candidates depending on Candidate 1's disutility of deviating from bliss point (δ_1). We set $\overline{p_2} = 0.25$, $\overline{p_1} = -0.25$, $\lambda_1 = \lambda_2 = 1$, $\delta_2 = 2$, and $\alpha = 1/3$. Budget constraints are not active.

Result 5 (Clientelism Level and Willingness to Pay). *Suppose $\delta_1 = \delta_2$ and $\lambda_1 = \lambda_2$.*

Then it is a sufficient condition that $\lambda_i \leq 1$ and $\delta_i \geq 1/4$ for the following to hold:

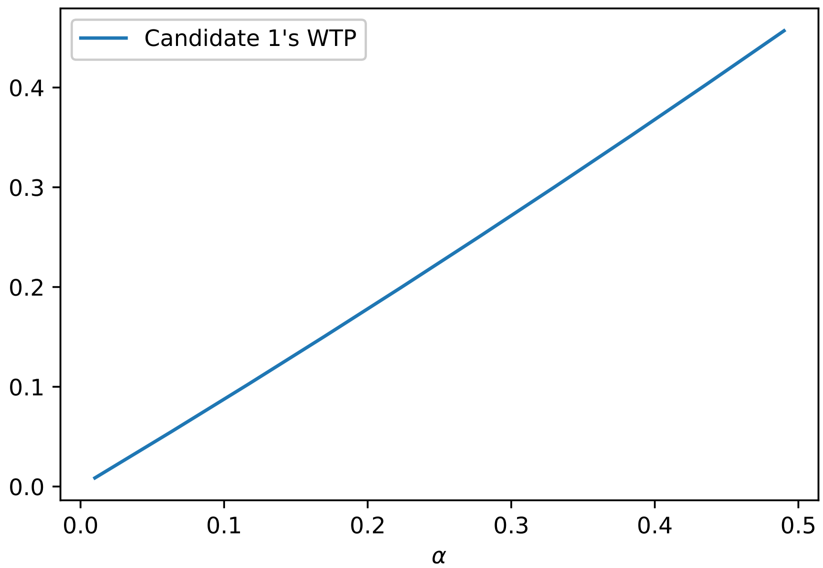
$$\frac{\partial \Phi_i}{\partial \alpha} \geq 0 \quad \forall \alpha \in [0,1]$$

□

Proof. See Appendix A.3.6. ■

As shown in Figure 6, as α approaches zero, the willingness to pay tends to zero. Conversely, a positive relationship exists between clientelistic votes and willingness to pay.

Figure 6: Clientelism level and willingness to pay for votes

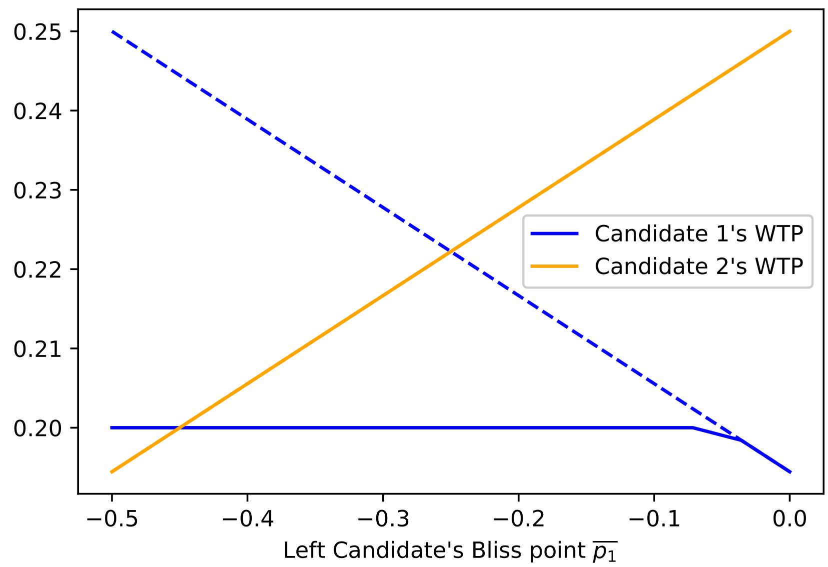


Notes: The figure plots the willingness to pay for votes of Candidate 1 depending on α. We set the same parameters as in Figure 3. Additionally, we set $\Phi_1 = \Phi_2$. Budget constraints are not active.

4.3 Interaction between Extremism and Resources

Let's now consider the case where budget constraints become active. According to Proposition 2, even if a candidate values vote-buying more, their resources could eventually prevent them from buying votes. In Figure 7, we observe that Candidate 1, despite valuing vote-buying, has an active constraint preventing them from accessing votes. Remember that we don't model how the candidate obtains resources; instead, we assume they are exogenous to the model.

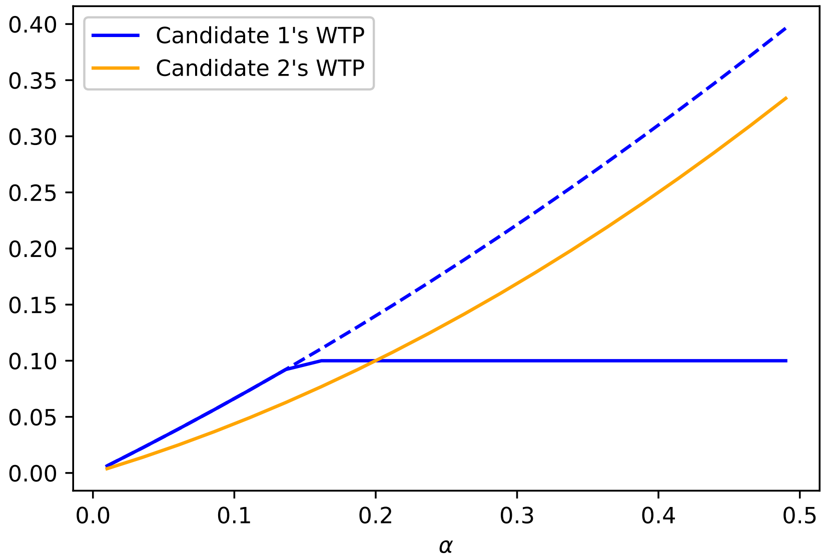
Figure 7: Willingness to pay for votes and budget constraints



Notes: The figure plots the willingness to pay for votes depending on Candidate 1's bliss point. The blue solid line represents Candidate 1's willingness to pay with an active budget constraint. The Dashed lines represent the willingness to pay without constraints of Candidate 1 (blue) and Candidate 2 (yellow). We set the same parameters as in Figure 3. Additionally, we set $R_1 = 0.2$.

Another interesting result is to observe what happens with changes in α. As shown in Figure 8, when the available votes from the broker are low, the willingness to pay of both candidates is low, implying that budget constraints are not active, so the more extreme candidate would buy the votes. However, as the proportion of clientelistic votes grows, budget constraints become active, causing Candidate 2 to purchase the votes despite being less extreme.

Figure 8: Willingness to pay for votes depending on α



Notes: The figure plots the willingness to pay for votes depending on the level of clientelism α. The blue solid line represents Candidate 1's willingness to pay with an active budget constraint. The Dashed lines represent the willingness to pay without constraints of Candidate 1 (blue) and Candidate 2 (yellow). We set the same parameters as in Figure 3. Additionally, we set $R_1 = 0.1$.

5 Extensions

5.1 The Case of Two Parties and General Election

Next, we consider the effect of external competition on the internal party election (primary election). The primary winner must face an external candidate from another party in the general election. We show that while the previous results hold, the effect of vote-buying is reduced.

To make the analysis tractable, we introduce two simplifying assumptions. First, we assume that the platform of the external candidate is exogenous and equal to the median voter in the distribution of general preferences. Second, we assume that the platforms chosen by the candidates in the primary cannot be changed in the general election. In other words, we rule out opportunistic behavior by the candidates. This assumption implies that there is a significant credibility cost associated with changing the policy proposal, which would deter such behavior.[17] Intuitively, there is a credibility cost of proposing something significantly different in the primary than in the general election. We consider the most extreme case where the platform proposed in the primary is a firm commitment.

Description:

Now consider that the candidate who wins the internal party election must compete in a general election against a candidate from another party. The new utility function is

$$\pi_i(\varphi_i, p_i, \beta_i) = \varphi_i + \varphi_i \varphi_i^g \cdot A - \delta_i \left(p_i - \overline{p_i}\right)^2 + m(R_i - \beta_i), \tag{14}$$

where equation (14) is identical to equation (1), except for $\varphi_i \varphi_i^g \cdot A$, where φ_i^g is the probability of winning the general election, dependent on the proposed platforms, and A is the payment received for winning this election. Note that equation (14) reflects the expected payment of the electoral process given φ and φ^g. In effect, neglecting the ideological loss function and assuming that the payment for winning the primary is A_0 (implicitly assumed to be $A_0 = 1$), the expected payment is

$$\varphi\left((1 - \varphi^g) \cdot A_0 + \varphi^g \cdot (A_0 + A)\right) + (1 - \varphi) \cdot 0 = \varphi A_0 + \varphi \varphi^g A.$$

In other words, the term $\varphi_i \cdot \varphi_i^g$ reflects the probability of winning both the primary and the general election.

Consider a party with the same characteristics as previously described. In the general election, voter preferences are distributed uniformly in the interval $[-1/2, 1/2]$.

The equation (14) becomes

$$\pi_i(\varphi_i, p_i, \beta_i) = \varphi_i + \varphi_i \cdot \left(\frac{p_i + p_{ex}}{2} + \frac{1}{2}\right) A - \delta_i \left(p_i - \overline{p_i}\right)^2 + m(R_i - \beta_i), \tag{15}$$

[17] While this cost might only *attenuate* the discrepancy between the platforms rather than necessarily eliminate it, we assume that this cost is high enough to prevent it completely.

where p_{ex} represents the platform proposed by the candidate from the other party in the external election. We assume that the choice of this platform always coincides with the median of preferences. Additionally, we assume that these preferences are uniformly distributed.

The reaction functions are[18]

$$\widehat{p_1}(p_2) = \left(\frac{0.25\,(1-\alpha)\,(1-\alpha\lambda_1)}{2\delta_1 - 0.5\,(1-\alpha)\,(1-\alpha\lambda_1)}\right) \cdot p_2 + \frac{\alpha^2\lambda_1 - \alpha\lambda_1 - 0.5\alpha + 2\delta_1\overline{p_1} + 1}{-0.5\alpha^2\lambda_1 + 0.5\alpha\lambda_1 + 0.5\alpha + 2\delta_1 - 0.5}$$
$$\widehat{p_2}(p_1) = \left(\frac{0.25\,(1-\alpha)}{0.5\alpha + 2\delta_2 - 0.5}\right) \cdot p_1 + \frac{\alpha + 2\delta_2\overline{p_2} - 1}{0.5\alpha + 2\delta_2 - 0.5}.$$

Then, the optimal platforms proposed by each candidate are

$$\begin{aligned}\widehat{p_1}(\alpha, 0) = {} & \frac{\delta_1\,(\alpha - 1 + 4\delta_2)}{M} \cdot \overline{p_1} + \frac{\delta_2\,((1-\alpha)(1-\lambda_1\alpha))}{2M} \cdot \overline{p_2} + \\ & + \frac{(3/4)\lambda\alpha^3 + (2\delta_2\lambda_1 - (3/2)\lambda - (1/2))\,\alpha^2}{M} + \\ & \frac{(-2\delta_2\lambda_1 - \delta_2 + (3/4)\lambda_1 + (3/4))\,\alpha + 2\delta_2 - 3/4}{M}\end{aligned}$$

$$\begin{aligned}\widehat{p_2}(\alpha, 0) = {} & \frac{\delta_1(1-\alpha)}{8M} \cdot \overline{p_1} + \frac{\alpha\delta_2(1 + \lambda_1(1-\alpha)) + 4\delta_2\delta_1 - \delta_2}{4M} \cdot \overline{p_2} - \\ & - \frac{(1-\alpha) + 1/32\alpha}{2M},\end{aligned}$$

where M is a constant that depends on the model's parameters, i.e., α, δ_1, δ_2, and λ_1.

Result 6 (Platforms Moderation). *Consider a general election where voter preferences are distributed according to $G \sim [-1/2, 1/2]$. Candidates within the party solve equation (15). Additionally, assume that the candidate who opposes the winner of the internal election always chooses to propose the distribution median, i.e., $p^{ex} = 0$. Then, it holds that $\forall \alpha \in [0, 1]$*

[18] For simplicity, we set $A = A_0 = 1$.

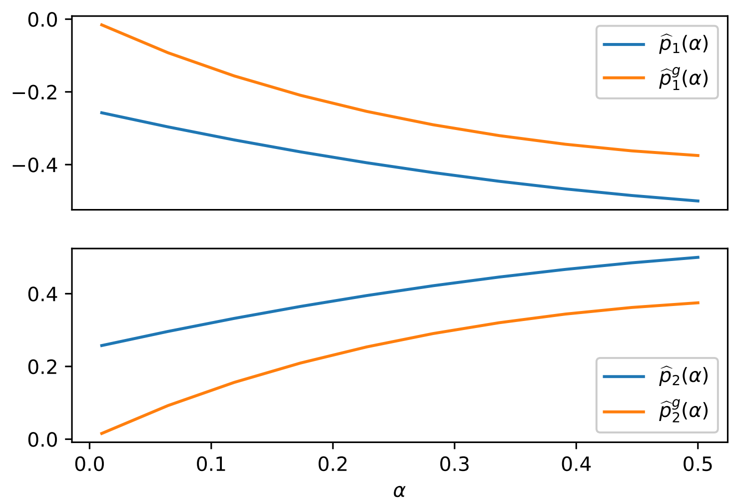
$$|\widehat{p_1}(\alpha,0)| > |\widehat{p_1}^g(\alpha,0)|$$
$$|\widehat{p_2}(0,\alpha)| > |\widehat{p_2}^g(0,\alpha)|$$

□

Proof. See Appendix A.3.7. ■

This result is reflected in Figure 9. As we can see, the platform in the presence of general elections is closer to the median than without general elections. However, despite this moderating effect, it still holds that a higher level of clientele votes implies a more significant divergence from the median.

Figure 9: Optimal platforms with general election



Notes: The figure plots optimal platforms depending on the level of clientelism α for both cases, with and without a general election. $\widehat{p_i}^g$ corresponds to the optimal platform of candidate i with general election while $\widehat{p_i}$ corresponds to the optimal platform of candidate i without general election (baseline case). We set the same parameters as in Figure 3.

One way to quantify this moderating effect on political platforms is to calculate the level of clientele militancy α_0 necessary for both platforms to be equal. In other words, for a level of clientele militancy α, find $\alpha_0(\alpha)$ such that

$$\widehat{p_1}(\alpha_0(\alpha), 0) = \widehat{p_1}^g(\alpha, 0). \tag{16}$$

Then, the ratio $\alpha(\alpha)/\alpha_0(\alpha)$ indicates how often the level of clientele militancy in the case with general elections needs to be compared to the case without general elections to make the platforms the same. The higher this ratio, the greater the moderating effect of the general election. Directly substituting equation (6) into equation (16), $\alpha_0(\alpha)$ is

$$\alpha_0(\alpha) = \frac{1+\lambda_1}{2\lambda_1} - \frac{\sqrt{(1+\lambda_1)^2 - 4 + 16\lambda_1\delta_1(\widehat{p_1}^g(\alpha, 0) - \overline{p_1})}}{2\lambda_1},$$

when $\lambda_1 > 0$, while if $\lambda_1 = 0$, $\alpha_0(\alpha)$ is

$$\alpha_0(\alpha) = 1 - 4\delta_1(\widehat{p_1}^g(\alpha, 0)).$$

Result 7 (Measurement of the Moderation Effect). *Consider the function $\alpha_0(\alpha)$ defined implicitly in equation (16). Then, it holds that*

$$\tfrac{d\alpha_0(\alpha)}{d\alpha} > 0$$

□

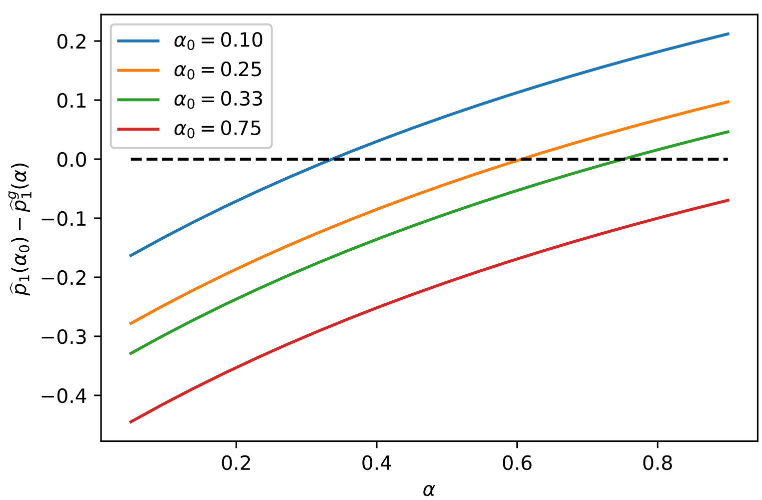
Proof. See Appendix A.3.8. ■

For instance, setting $\overline{p_1} = -\overline{p_2} = -1/2$ and $\delta_1 = \delta_2 = 2$, α_0 can be approximated by[19]

$$\alpha_0(\alpha) \approx 0.87\alpha - 0.43 \text{ if } \lambda_1 = 0$$
$$\alpha_0(\alpha) \approx 0.41\alpha + 0.02 \text{ if } \lambda_1 = 1$$
$$\alpha_0(\alpha) \approx 0.33\alpha + 0.19 \text{ if } \lambda_1 = 2.$$

This implies, for example, that if $\lambda = 1$ and $\alpha_0 = 30\%$, a level of clientelism around $\alpha \approx 66\%$ would be needed for the platform to be equally extreme in a context with general elections. This result is illustrated in Figure 10. The higher the level of clientelism α_0 in a context without general elections, the higher the level of clientelism α needs to be in the context with general elections.

Figure 10: Moderating effect of general elections



Notes: The figure plots the difference on optimal platforms with and without general election $\widehat{p}_1(\alpha_0) - \widehat{p}_1^g(\alpha)$ depending on the level of clientelism α. The dashed line indicates the points where $\widehat{p}_1(\alpha_0) = \widehat{p}_1^g(\alpha)$. We set the same parameters as in Figure 3.

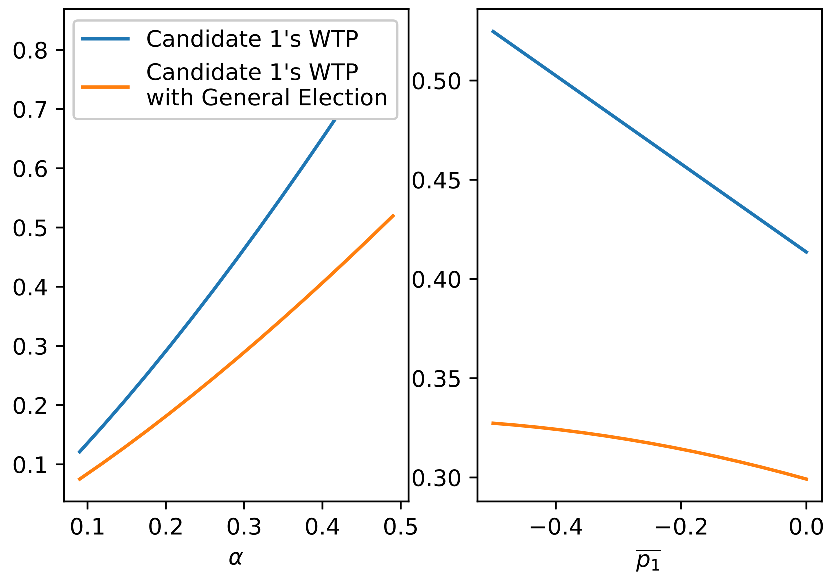
One final analysis to illustrate the effect of external elections on the dynamics of vote-buying is to compare the broker's willingness to pay for votes between the original scenario with only internal polls and the scenario with general elections. First, it's

[19]This corresponds to a first-order Taylor approximation around $\overline{\alpha} = 0.5$. See details in Appendix A.3.8.

important to note that these willingness-to-pay values cannot be directly compared, as the scenario with internal elections includes the expected benefit of winning both the general and primary elections. Consequently, the willingness to pay is always higher in this case.

To address this issue, we compare a scenario where the internal election perfectly determines the general election outcome with a scenario where the general election is competitive. As shown in Figure 11, the presence of a competitive general election reduces the local leader's willingness to pay for votes.

Figure 11: Willingness to pay with general elections



Notes: The figure plots the willingness to pay for votes of Candidate 1 depending on α (left panel) and $\widehat{p_1}$ (right panel). The yellow line corresponds to the case with a general election, while the blue line corresponds to the case without a general election. The left panel sets $\overline{p_1} = -0.5$ and $\overline{p_2} = 0.5$ while the right panel sets $\alpha = 0.33$ and $\overline{p_2} = 0.5$. We set the same parameters as in Figure 3. No budget constraints are considered.

5.2 Alternative Modeling: Maximizing the Probability of Winning

Until now, we have not incorporated the probability of a candidate winning into our model. Instead, we have used the share or the margin of votes. However, these measures are generally not equivalent (*win motivation vs. vote motivation*, see Duggan

(2005)). Furthermore, in the presence of the ideological preferences of the candidates, additional conditions are required for these two measures to generate equivalent results. Deriving general results for cases where the probability of winning is maximized is more complex than for the case of vote maximization. While it is true that there are contexts where vote maximization is more appropriate, such as in legislative elections, we present a model below that calculates the probability of victory.

Similar to the original model, we have a group of ideological and clientelistic voters. In the case of clientelistic voters, they vote for the candidate who buys their votes. For ideological voters, we consider the probabilistic voting scheme used in Kamada and Kojima (2013) and Kamada and Kojima (2014). In this model, each voter i receives the following levels of utility depending on whether Candidate 1 or Candidate 2 wins

$$u_i^1 = -(p_i - p_1)^2 + \eta$$
$$u_i^2 = -(p_i - p_2)^2,$$

where p_i represents the bliss point of voter i, and p_1 and p_2 are the proposed platforms of candidates 1 and 2.[20] Moreover, η represents a general shock to preferences. This value of η is known as the valence of the candidate, reflecting relevant candidate characteristics independent of the proposed political platform. These characteristics could include perceived conflict-handling abilities, charisma, or even positive or negative incidents that arise during the campaign. Note that η does not necessarily need to be ≥ 0, as a negative value can be interpreted as a positive valence of the other candidate.[21] We assume that $\eta \sim G$ with a mean of zero and that voter preferences follow the distribution $p \sim F$.

Voter i votes for Candidate 1 whenever $u_i^1 \geq u_i^2$. Therefore, the voter indifferent between both candidates has preferences p such that

[20]There is a slight notational abuse since p_1 and p_2 only represent the candidates, although p_i represents a voter.

[21]This model is equivalent to a model with a different shock for each candidate, η_1 and η_2, where $\eta \equiv \eta_1 - \eta_2$

$$-(p-p_1)^2+\eta=-(p-p_2)^2$$
$$p^*=\frac{p_1+p_2}{2}+\frac{\eta}{2(p_2-p_1)}=p^m+\frac{\eta}{2(p_2-p_1)}.$$

As a result, the number of votes obtained by Candidate 1 from the ideological group is

$$\int_{-\infty}^{p^*} dF(p)=F\left(p^*\right)-F(-\infty)=F\left(p^*\right).$$

Note that if $\eta=0$, then $p^*=(p_A+p_B)/2=p^m$, making the total ideological group votes the same as in our original model. Therefore, the share of total votes that Candidate 1 obtains when buying the broker's votes is

$$V_1=\alpha+(1-\alpha)F\left(p^*\right).$$

Subsequently, the probability of Candidate 1 winning is

$$\begin{aligned}
\Pr\left(V_1(\alpha,0)\geq\frac{1}{2}\right)&=\Pr\left(F(p^*)\geq\frac{1-2\alpha}{2(1-\alpha)}\right)=\Pr\left(p^*\geq F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right)\right)\\
&=\Pr\left(\frac{p_1+p_2}{2}+\frac{\eta}{2(p_2-p_1)}\geq F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right)\right)\\
&=\Pr\left(\eta\geq 2\left(F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right)-p^m\right)(p_2-p_1)\right)\\
&=1-G\left(2\left(F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right)-p^m\right)(p_2-p_1)\right). \qquad (17)
\end{aligned}$$

Some direct conclusions from the previous expression are worth noting. If $\alpha=0$, meaning there is no clientelistic militancy, we see that

$$F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right) = F^{-1}(1/2) = 0.$$

Hence, for equal or symmetric platforms around the median, we have

$$\Pr\left(V_1 \geq \frac{1}{2}\right) = \Pr\left(V_2 \geq \frac{1}{2}\right) = \frac{1}{2}.$$

On the other hand, if $\alpha = 1/2$, meaning that clientelistic militancy is half of the total militancy, we have

$$F^{-1}(0) = -\infty.$$

Therefore,

$$\Pr\left(V_1(1/2, 0)\right) = 1 - G(-\infty) = 1.$$

This result is highly intuitive, as by buying at least half of the potential voters, the probability of winning the election tends towards 100%, as only a single ideological voter needs to vote for this candidate to surpass 50% of the votes. The baseline model does not capture this feature.

The optimization problems of both candidates are

$$\max_{p_1} \Pr\left(V_1(\alpha, 0) \geq \frac{1}{2}\right) - \delta_1(p_1 - \overline{p_1})^2$$
$$\max_{p_1} \left(1 - G\left(2\left(A(\alpha) - p^m\right)\left(p_B - p_A\right)\right)\right) - \delta_1(p_A - \overline{p_A})^2$$
$$\max_{p_2} \left(1 - \Pr\left(V_1(\alpha, 0) \geq \frac{1}{2}\right)\right) - \delta_2(p_1 - \overline{p_1})^2$$
$$\max_{p_2} \left(G\left(2\left(A(\alpha) - p^m\right)\left(p_B - p_A\right)\right)\right) - \delta_2(p_A - \overline{p_A})^2,$$

where $A(\alpha) \equiv F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right)$, and $\overline{p_1}$ and $\overline{p_2}$ correspond to the bliss points of the candidates.

Proposition 4. *Let $\widehat{p_1}(\alpha, 0)$ and $\widehat{p_2}(0, \alpha)$ be the optimal platforms of the candidates. Then, it is a sufficient condition that*[22]

$$G''\left(2\left(A(\alpha) - p^m\right)\left(\widehat{p_2} - \widehat{p_1}\right)\right) > 0$$

for the optimal platform of the candidate who won the votes to diverge from the median with increases in α, i.e.,

$$\frac{d\widehat{p_1}(\alpha, 0)}{d\alpha} \leq 0 \ \ and \ \ \frac{d\widehat{p_2}(0, \alpha)}{d\alpha} \geq 0.$$

For the platforms of candidates who do not buy the votes to diverge, i.e.,

$$\frac{d\widehat{p_1}(0, \alpha)}{d\alpha} \leq 0 \ \ and \ \ \frac{d\widehat{p_2}(\alpha, 0)}{d\alpha} \geq 0,$$

it is a sufficient condition that

$$\frac{-\delta - 2G'}{G'} < \frac{G''}{G'} < \frac{1}{2(\widehat{p_2} - \widehat{p_1})(\widehat{p_1} - A(\alpha))}$$

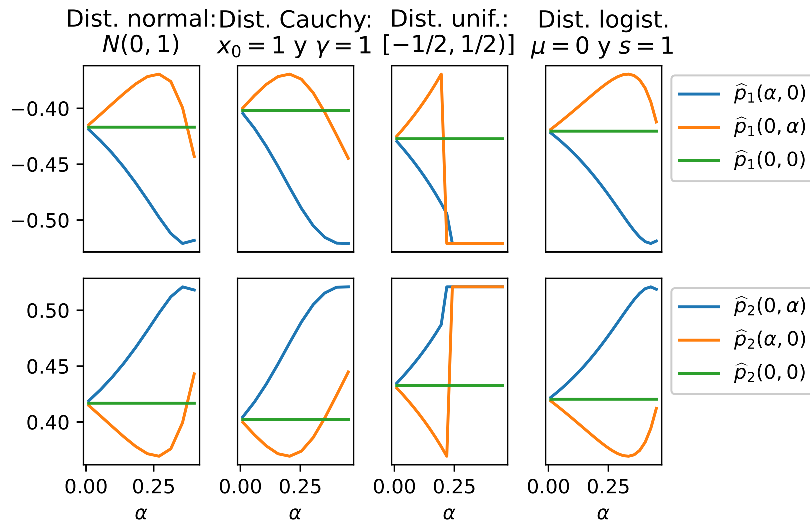
where G' and G'' are evaluated at $\left(2\left(A(\alpha) - p^m\right)\left(\widehat{p_2} - \widehat{p_1}\right)\right)$. □

The above proposition is illustrated in Figure 12. We can observe that the candidate who buys the votes gets closer to their bliss point when buying votes. Additionally, this effect grows as α increases.

Concerning the candidate who does not buy the votes, we observe that initially, they tend to move closer to the party median. However, when α is sufficiently large,

[22]This is, G'' evaluated at the critical point given in equation (17).

Figure 12: Optimal platforms in maximizing the probability of winning



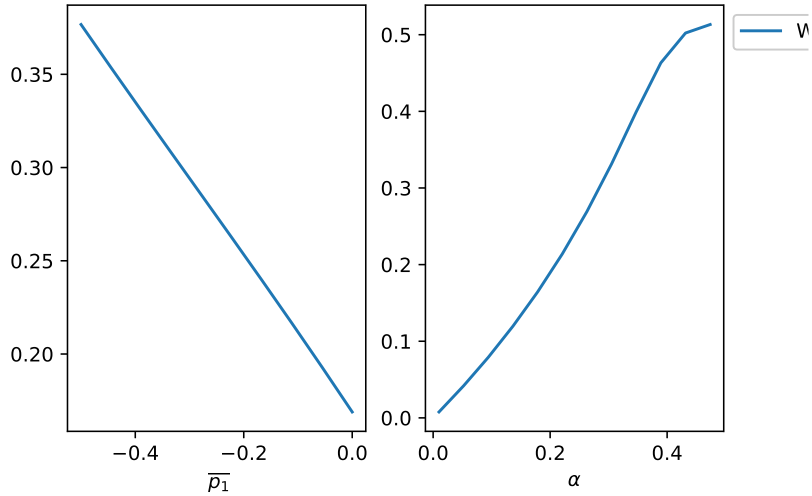
Notes: The figure plots optimal platforms depending on the level of clientelism α for different distributions of the shock of preferences η. The distribution of the militancy preferences is $F \sim N(0, 1)$. We set the same parameters as in Figure 3.

they move closer to their bliss point. This result is because the weight of ideological militancy has decreased enough. As a result, the candidate finds it more profitable to reduce the penalty for deviating from their preference rather than gaining votes in the ideological group.

An interesting case is when the shock of preferences is uniformly distributed. In this case, if α is large enough, the candidates abruptly move to their preferred platforms, as at that level of α, the probability of winning for the candidate who bought the votes is 100%.

Finally, let's examine what happens to the willingness to pay for votes in this new context. As seen in Figure 13, the willingness to pay increases with greater extremism of the candidate and increases with the level of party clientelism.

Figure 13: Willingness to pay in maximizing the probability of winning



Notes: The figure plots the willingness to pay for votes of Candidate 1 depending on α (right panel) and $\widehat{p_1}$ (left panel). The distribution of the shock η is $G \sim N(0,1)$. The distribution of the militancy preferences is $F \sim N(0,1)$. For the simulation we set $\overline{p_1} = -0.5$, $\overline{p_2} = 0.5$ and $\delta_1 = \delta_2 = 2$.

6 Conclusion

This research turns into whether clientelism influences the representative capacity of political parties. Through a model, we show that clientelism accentuates the divergence of political platforms to the median preferences of the political party, which reduces the representation of militancy. Furthermore, we show that those candidates with ideological preferences further from the party median are more willing to use clientelistic networks. The possible abstention of non-clientel militancy reinforces this divergence. We show that vote-buying diverges the platform of those who buy votes and allows candidates who do not use clientelistic networks to move their platforms away from the median since non-clientele militancy has a lower relative weight on the total votes. On the other hand, general elections make it possible to significantly moderate this effect, limiting the divergence to the median and reducing the value that candidates give to buying votes.

Finally, this work suggests different mechanisms to alter the effects of clientelism. On the one hand, party competition can significantly limit the results predicted by this model. In this sense, strengthening primaries between parties could reduce the distorting effects of the presence of clientelistic voters. Another mechanism is to increase transparency in the party's internal processes so that the use of clientelistic networks is noticeable to ideological voters. Regarding our model, the latter will increase the penalty for buying votes, which, as our model also shows, can completely discourage vote-buying.

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A Appendix

A.1 Robustness Exercise: Extension to Different Party Preference Distributions

So far, we have assumed that political preferences within the party are uniformly distributed between $\left[\underline{p}, \overline{p}\right]$. Instead, suppose these preferences follow the distribution $F(x)$ with probability density function $f(x)$.[23] Under this assumption, the first-order condition becomes

$$1/2f\left(p_m\right) \cdot \left((1-\alpha)(1-\lambda_1\alpha)\right) - 2\delta\left(p_1 - \overline{p_1}\right) = 0, \tag{18}$$

where $p_m = \frac{p_1+p_2}{2}$. We use the implicit function theorem to replicate the comparative statics conducted throughout this work. Remember that this theorem states, informally, that for a function $\Phi : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\Phi(x, y) = 0,$$

it follows that

$$\frac{dy}{dx} = -\frac{\frac{\partial\Phi}{\partial x}}{\frac{\partial\Phi}{\partial y}}.$$

In this case, we take

$$\Phi(p_1, p_2, \alpha_1, \alpha_2, \alpha) = 1/2f\left(p_m\right) \cdot \left((1-\alpha)(1-\lambda_1\alpha)\right) - 2\delta\left(p_1 - \overline{p_1}\right) = 0.$$

Then, we see that

$$\frac{\partial\Phi}{\partial p_1} = 1/4(1-\alpha)(1-\alpha\lambda_1)f'(p_m) - 2\delta$$

[23] This corresponds to the more general case of random utility described by Banks and Duggan (2005).

$$\frac{\partial \Phi}{\partial \alpha_1} = \left(2\alpha^2\lambda_1 - 1 - \lambda_1\right) f(p_m)\frac{1}{2} < 0.$$

Thus, the result known as the **broker effect** depends on whether the following condition holds

$$\frac{dp_1}{d\alpha_1} = f(p^m)\frac{\left(-2\alpha^2\lambda_1 + 1 + \lambda_1\right)}{1/4(1-\alpha)(1-\lambda_1\alpha)f'(p^m) - 2\delta_1} < 0. \tag{19}$$

A sufficient condition for the broker effect to hold is that $f'(p_m) \leq 0$. This is satisfied for a p_m to the right of the median (in any single-peaked distribution). For example, for a uniform distribution $f' = 0$, the result holds. Another case is when the candidates are symmetric. In this case, p^m coincides with the party's median, so again, $f' = 0$. In the case where $f' > 0$, it is required that

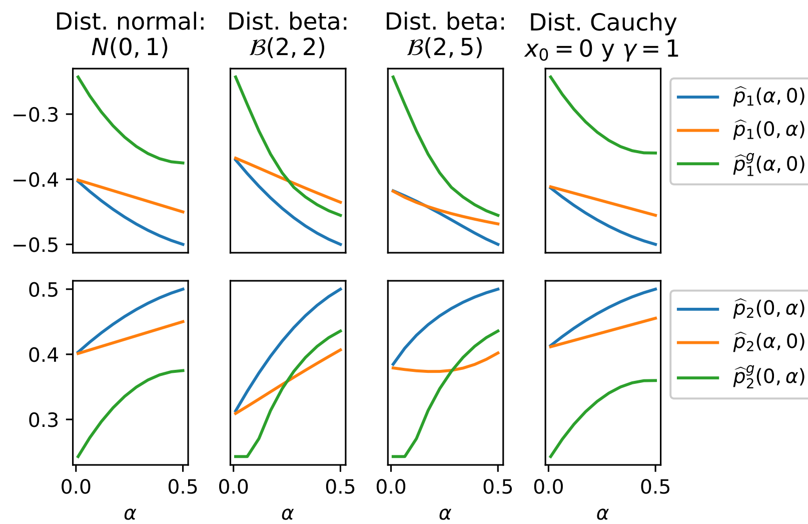
$$(1-\alpha)(1-\lambda_1\alpha)f'(p^m) < 2\delta_1.$$

Applying the implicit function theorem, we obtain that the result known as **complementary corruption** holds as long as

$$\frac{dp_1}{d\alpha} = f(p_m)\frac{1}{1/2(1-\alpha)f'(p_m) - 2\delta} < 0. \tag{20}$$

In Figure A.1, we can observe the behavior of the model's results under different distributions. The increase α always pushes the platforms away from the median, even if the candidate doesn't win the broker's votes. Figure A.1 also illustrates the effect of general elections in this context. Similar to the model with uniform distribution, the platforms during general elections are the closest to the median.

Figure A.1: Optimal Platforms with Alternative Distributions



Notes: The figure plots optimal platforms for both candidates depending on the level of clientelism α for different distributions of the political preferences within the party. $\widehat{p_i}^g$ corresponds to the optimal platform of candidate i during general elections. For the simulation we set $\overline{p_1} = -0.5$, $\overline{p_2} = 0.5$.

A.2 Discussion on the Penalty for Using Clientelist Votes

Throughout this paper, we have assumed that candidates incur a penalty λ_i for purchasing a quantity α of clientelist votes. We have indicated that this parameter reflects an aversion on the part of the ideological membership to vote for a candidate who buys votes.

The effect on the share of voters is given by

$$(1 - \lambda_i \alpha)(1 - \alpha)F(p^m).$$

Here, $(1 - \alpha)F(p^m)$ represents the proportion of votes that a candidate would receive without buying votes. The penalty factor $(1 - \lambda_i \alpha)$ can be rationalized by assuming that each voter gets a signal that candidate i bought votes with a probability $h(\alpha)$, where α denotes the number of votes purchased. It's reasonable to think that $h' > 0$, as a higher quantity of purchased votes, would more likely lead to this fact becoming known.

Each voter who receives the signal with probability $h(\alpha)$ will not vote for candidate i with a probability of λ_i. Consequently, the probability that a randomly selected voter does not vote for candidate i is $h(\alpha)\lambda_i$. Therefore, the proportion of votes that candidate i would receive is

$$(1 - h(\alpha)\lambda_i)(1 - \alpha)F(p^m).$$

In the case where h is a linear function, we get

$$(1 - \alpha\lambda_i)(1 - \alpha)F(p^m).$$

A.3 Appendix Proofs

A.3.1 Proposition 1

Suppose that Candidate 1 won the votes in the first stage. The optimal platform of Candidate 1 follows from

$$\max_{p_1} \alpha + (1-\alpha)(1-\lambda_1\alpha)F(p^m) - \delta_1(p_1 - \overline{p_1})^2.$$

Differentiating with respect to p_1 and recalling that $F(p^m) = p^m + 1/2$ we have

$$\begin{aligned} \frac{1}{2}(1-\alpha)(1-\lambda_1\alpha) - 2\delta_1(p_1 - \overline{p_1}) &= 0 \\ p_1 - \overline{p_1} &= \frac{(1-\alpha)(1-\lambda_1\alpha)}{4\delta_1}, \end{aligned}$$

rearranging we get equation (6). The optimal platform of Candidate 2 follows from

$$\max_{p_2} (1-\alpha)(1-F(p^m)) - \delta_2(p_2 - \overline{p_2})^2,$$

where the FOC is

$$\begin{aligned} -\frac{1}{2}(1-\alpha) - 2\delta_2(p_2 - \overline{p_2}) &= 0 \\ (p_2 - \overline{p_2}) &= -\frac{(1-\alpha)}{4\delta_2}, \end{aligned}$$

rearranging we get equation (7). On the other hand, if the votes are bought by Candidate 2, we have that Candidate 1 and Candidate 2 solve, respectively, the following problems

$$\max_{p_1}(1-\alpha)F(p^m) - \delta_1(p_1 - \overline{p_1})^2$$

$$\max_{p_2}\alpha + (1-\alpha)(1-\lambda_2)(1-F(p^m)) - \delta_2(p_2\overline{p_2})^2,$$

where the FOCs are, respectively,

$$-\frac{1}{2}(1-\alpha)(1-\lambda_2\alpha) - 2\delta_2(p_2 - \overline{p_2}) = 0$$
$$\frac{1}{2}(1-\alpha) - 2\delta_1(p_1 - \overline{p_1}) = 0,$$

rearranging we get equations (8) and (9).

Finally, if we assume that no candidate buys the votes, each candidate solves

$$\max_{p_1} F(p^m) - \delta_1(p_1 - \overline{p_1})^2$$

$$\max_{p_2}(1 - F(p^m)) - \delta_2(p_2\overline{p_2})^2$$

FOCs

$$-\frac{1}{2} - 2\delta_2(p_2 - \overline{p_2}) = 0$$
$$\frac{1}{2} - 2\delta_1(p_1 - \overline{p_1}) = 0$$

from where we get equations (10) and (11),

Now, we verify the second-order condition. Indeed,

$$\frac{\partial^2 \pi_i(\widehat{p_1}, \widehat{p_2})}{\partial p_i^2} = -2\delta_i \leq 0.$$

A.3.2 Proposition 2

In this section, we prove that the willingness to pay for buying votes is

$$\Phi_1 = \widehat{\pi}_1(\alpha, 0) - \widehat{\pi}_1(0, \alpha) = \pi_{11} - \pi_{12}$$
$$\Phi_2 = \widehat{\pi}_2(0, \alpha) - \widehat{\pi}_2(\alpha, 0) = \pi_{22} - \pi_{21},$$

where $\widehat{\pi}_i\,(\alpha_1, \alpha_2) \equiv \pi_i(\widehat{p_1}(\alpha_1, \alpha_2), \widehat{p_2}(\alpha_1, \alpha_2))$, and $\widehat{p_i}(\alpha_1, \alpha_2)$ is the optimal platform of candidate i conditional to (α_1, α_2).

First, we prove that there is no equilibrium where both candidates abstain. Indeed, since $\pi_{ii} > \pi_{i0}\ \ \forall i$, any candidate could deviate and go from winning π_{i0} to π_{ii} .

Now we prove that the disposition of each candidate i is $\pi_{ii} - \pi_{ij}$. If Candidate 1 does not buy votes, Candidate 2 will buy them at any price since we have proven that there will always be incentives to buy them. Therefore, the utility of Candidate 1 would be equivalent to the utility received from competing without clientelist votes against Candidate 2, who has all of the broker's votes: π_{12}. On the other hand, if he manages to buy the votes, his utility will be π_{11}. Therefore, his willingness to pay will be $\Phi_1 = \pi_{11} - \pi_{12}$. The same argument applies to Candidate 2.

Finally, we prove that the equilibrium supply is $\beta_i^* = \min\{\Phi_i, R_i\}$. Remember that this auction's design corresponds to a second-price auction. This is a known result in auction theory (see Krishna (2009)). If the willingness to pay is less than the budget, i.e., $\Phi_i \leq R_i$, then the restriction is not active, and the same reasoning can be applied as for the second price auction with $\beta_i^* = \Phi_i$.

On the other hand, if $\Phi > R_i$, we should note that bidding $\tilde{\beta}_i > R_i$ is a dominated strategy. If you win the auction with $\tilde{\beta}_i$ and the second best bid is less than R_i, you could have won by bidding R_i. If the second best bid exceeds R_i, the bid $\tilde{\beta}_i$ should be withdrawn. Therefore, bid $\beta_i^* = R_i$. Then, $\beta_i^* = \min\{\Phi_i, R_i\}$.

A.3.3 Proposition 3

This section proves the cases stated in the proposition.

Case 1:

Since both candidates abstain from participating, their payoffs are (π_{10}, π_{20}). If candidate i deviates and participates in the auction, they obtain π_{ii} since the other candidate is not participating. However, by hypothesis, π_{ii} is less than π_{i0}. Therefore, no candidate deviates from this equilibrium.

■

Case 2:

In this case, there are three equilibria. The first equilibrium is abstention. Its proof follows the same logic as Case 1. Next, we demonstrate that there is equilibrium with participation. Let us remember that in this equilibrium, the offers are

$$\beta_1 = \min\{\Phi_1', R_1\} \text{ and } \beta_2 = \min\{\Phi_2', R_2\}.$$

By reasoning identical to the proof of proposition 2, we see that bidding other than Φ' is a dominated strategy.

Next, we show the existence of a mixed strategy. Let us remember that, for an equilibrium $(\sigma_1, 1-\sigma_1, \sigma_2, 1-\sigma_2)$, it must be true that each player is indifferent between any of the possible strategies. For Candidate 1, we have

$$\mathbb{E}\left(\pi_{1\bullet} | \beta_1 = \phi\right) = \sigma_2 \pi_{10} + (1-\sigma_2)\pi_{12}$$
$$\mathbb{E}\left(\pi_{1\bullet} | \beta_1 \neq \phi\right) = \sigma_2 \pi_{11} + (1-\sigma_2)\mathbb{E}.(\pi_{1\bullet})$$

Solving for σ_2:

$$\sigma_2 = \frac{\mathbb{E}(\pi_{1\bullet}) - \pi_{12}}{\pi_{10} - \pi_{11} + \mathbb{E}(\pi_{1\bullet}) - \pi_{12}}.$$

Note that by hypothesis $\pi_{10} > \pi_{11}$. On the other hand, $\mathbb{E}(\pi_{1\bullet}) = \mu\pi_{11} + (1-\mu)\pi_{12}$. Then,

$$\begin{aligned} \mathbb{E}(\pi_{1\bullet}) = \mu\pi_{11} + (1-\mu)\pi_{12} &> \pi_{12} \\ \mu(\pi_{11} - \pi_{12}) &> 0, \end{aligned}$$

which is truce since $\pi_{11} - \pi_{12} > 0$ and $\mu > 0$. $\mathbb{E}(\pi_{1\bullet}) > \pi_{12}$. Consequently, $\sigma_2 > 0$. Now, we prove that $\sigma_2 < 1$. Note that $\sigma_2 < 1$ is equivalent to

$$\begin{aligned} &\mathbb{E}(\pi_{1\bullet}) - \pi_{12} < \pi_{10} - \pi_{11} + \mathbb{E}(\pi_{1\bullet}) - \pi_{12} \\ &\pi_{12} < \pi_{10}, \end{aligned}$$

which is also fulfilled by hypothesis. Consequently, $0 < \sigma_2 < 1$. For the case of σ_1, we proceed similarly, proving that $0 < \sigma_1 < 1$. ■

A.3.4 Results 1, 2, and 3

We have

$$\begin{aligned} \widehat{p_1}(\alpha, 0) - \widehat{p_1}(0,0) &= \frac{(1-\alpha)(1-\lambda_1) - 1}{4\delta_1} \leq 0 \\ \widehat{p_2}(0,\alpha) - \widehat{p_2}(0,0) &= \frac{1 - (1-\alpha)(1-\lambda_2)}{4\delta_2} \geq 0, \end{aligned}$$

since $1-\alpha$ and $1-\lambda_i\alpha$ are contain in the interval $[0,1]$. From the two previous inequalities we have that $|\widehat{p_1}(\alpha,0) \geq \widehat{p_1}(0,0)|$ and $|\widehat{p_2}(0,\alpha) \geq \widehat{p_2}(0,0)|$, which proves the result 1.

On the other hand,

$$\widehat{p_1}(0,\alpha) - \widehat{p_1}(0,0) = \frac{(1-\alpha)-1}{4\delta_1} \leq 0$$
$$\widehat{p_2}(\alpha,0) - \widehat{p_2}(0,0) = \frac{1-(1-\alpha)}{4\delta_2} \geq 0,$$

implying $|\widehat{p_1}(0,\alpha) \geq \widehat{p_1}(0,0)|$ and $|\widehat{p_2}(\alpha,0) \geq \widehat{p_2}(0,0)|$, which proves the result 2.

Finally, derivatives of equations (6), (7), (8) and (9), get the result 3.

A.3.5 Result 4

From equation (13)

$$\Phi_1 = (V_1(\alpha,0) - V_1(0,\alpha)) - \delta_1 \cdot (\widehat{p_1}(\alpha,0) - \overline{p_1})^2 + \delta_1 \cdot (\widehat{p_1}(0,\alpha) - \overline{p_1})^2$$

$$\Phi_2 = (V_2(0,\alpha) - V_2(\alpha,0)) - \delta_2 \cdot (\widehat{p_2}(0,\alpha) - \overline{p_2})^2 + \delta_2 \cdot (\widehat{p_2}(\alpha,0) - \overline{p_2})^2.$$

Then, Candidate 1 (resp. Candidate 2) wins the votes if $\Phi_1 - \Phi_2 \geq 0$ (resp. $\Phi_2 - \Phi_1 \geq 0$). Therefore, we solve for $\Phi_1 - \Phi_2$ to see how this difference depends on the model parameters.

First, we simplify the terms $\widehat{p_1}(\alpha,0) - \overline{p_1})^2 - (\widehat{p_1}(0,\alpha) - \overline{p_1})^2$ and $\widehat{p_2}(0,\alpha) - \overline{p_2})^2 - (\widehat{p_2}(\alpha,0) - \overline{p_2})^2$. Indeed,

$$\begin{aligned}
&(\widehat{p_1}(\alpha,0) - \overline{p_1})^2 - (\widehat{p_1}(0,\alpha) - \overline{p_1})^2 = \\
&\widehat{p_1}(\alpha,0)^2 - 2\widehat{p_1}(\alpha,0) + \overline{p_1}^2 - \left(\widehat{p_1}(0,\alpha)^2 - 2\widehat{p_1}(\alpha,0) + \overline{p_1}^2\right) = \\
&\left(\widehat{p_1}(\alpha,0)^2 - \widehat{p_1}(0,\alpha)^2\right) - 2\overline{p_1}\left(\widehat{p_1}(\alpha,0) - \widehat{p_1}(0,\alpha)\right) = \\
&(\widehat{p_1}(\alpha,0) + \widehat{p_1}(0,\alpha)) \cdot (\widehat{p_1}(\alpha,0) - \widehat{p_1}(0,\alpha)) - 2\overline{p_1}\left(\widehat{p_1}(\alpha,0) - \widehat{p_1}(0,\alpha)\right) = \\
&(\widehat{p_1}(\alpha,0) + \widehat{p_1}(0,\alpha) - 2\overline{p_1}) \cdot (\widehat{p_1}(\alpha,0) - \widehat{p_1}(0,\alpha))
\end{aligned}$$

Therefore

$$(\widehat{p_1}(\alpha,0)-\overline{p_1})^2-(\widehat{p_1}(0,\alpha)-\overline{p_1})^2=-\frac{(1-\alpha)^2}{16\delta_1^2}(2-\lambda_1\alpha)\lambda_1\alpha. \tag{21}$$

Similarly we obtain

$$(\widehat{p_2}(0,\alpha)-\overline{p_2})^2-(\widehat{p_2}(\alpha,0)-\overline{p_2})^2=-\frac{(1-\alpha)^2}{16\delta_2^2}(2-\lambda_2\alpha)\lambda_2\alpha. \tag{22}$$

Then, the difference between equations (21) and (22) is

$$\frac{(1-\alpha)^2}{16\delta^2}\left(\lambda_1-\lambda_2\right)\left(2\alpha-\alpha^2+\lambda_1+\lambda_2\right).$$

Next, we simplify $V_1(\alpha,0)-V_1(0,\alpha)$ and $V_2(0,\alpha)-V_2(\alpha,0)$.

$$\begin{aligned}
V_1(\alpha,0)-V_1(0,\alpha) &= \alpha+(1-\alpha)(1-\lambda_1\alpha)F(p^{m_1})-(1-\alpha)F(p^{m_2}) \\
&= \alpha+(1-\alpha)\left((1-\lambda_1\alpha)F(p^{m_1})-F(p^{m_2})\right) \\
&= \alpha+(1-\alpha)\left((1-\lambda_1\alpha)\left(p^{m_1}+\frac{1}{2}\right)-\left(p^{m_2}+\frac{1}{2}\right)\right) \\
&= \alpha+(1-\alpha)\left(p^{m_1}-p^{m_2}-\lambda_1\alpha\left(p^{m_1}-\frac{1}{2}\right)\right) \qquad (23)
\end{aligned}$$

$$\begin{aligned}
V_2(0,\alpha)-V_2(\alpha,0) &= \alpha-(1-\alpha)(1-\lambda_2\alpha)(1-F(p^{m_2}))-(1-\alpha)(1-F(p^{m_1})) \\
&= \alpha+(1-\alpha)\left(1-F(p^{m_2})-(1-F(p^{m_1}))-\lambda_2\alpha(1-F(p^{m_2}))\right) \\
&= \alpha+(1-\alpha)\left(p^{m_1}-p^{m_2}-\lambda_2\alpha\left(\frac{1}{2}-p^{m_2}\right)\right) \qquad (24)
\end{aligned}$$

Now we subtract the expressions (23) and (24) and we have

$$V_1(\alpha,0) - V_1(0,\alpha) - (V_2(0,\alpha) - V_2(\alpha,0)) = \alpha(1-\alpha)\left(-\lambda_1 p^{m_1} - \lambda_2 p^{m_2} + \frac{\lambda_2 - \lambda_1}{2}\right) \tag{25}$$

where $p^{m_1} = \left(\frac{\overline{p_1}+\overline{p_2}}{2} + \frac{1-\alpha}{2}\left(\frac{1-\lambda_1\alpha}{4\delta_1} - \frac{1}{4\delta_2}\right)\right)$ and $p^{m_2} = \left(\frac{\overline{p_1}+\overline{p_2}}{2} + \frac{1-\alpha}{2}\left(\frac{1}{4\delta_1} - \frac{1-\lambda_2\alpha}{4\delta_2}\right)\right)$.

Finally, setting $\delta_1 = \delta_2$ and $\lambda_1 = \lambda_2$, the difference between equations (21) and (22) is zero, while the equation (25) becomes

$$-\alpha(1-\alpha)\lambda(\overline{p_1} + \overline{p_2}).$$

Consequently $\Phi_1 - \Phi_2 \geq 0 \iff -\alpha(1-\alpha)(\overline{p_1}+\overline{p_2}) \geq 0 \iff \overline{p_2} \leq -\overline{p_1}$. But since $\overline{p_1} \leq 0$, this is equivalent to

$$|\overline{p_2}| \leq |\overline{p_1}|.$$

A.3.6 Result 5

Note that if $\delta_1 = \delta_2$ and $\lambda_1 = \lambda_2$, then $p^{m_1} = p^{m_2}$. Then, we have that the equation (23) reduces to

$$V_1(\alpha,0) - V_1(0,\alpha) = \alpha - \lambda_1\alpha(1-\alpha)\left(p^{m_1} + \frac{1}{2}\right)$$

We derive the previous expression, and we have

$$
\begin{aligned}
\frac{\partial\left(V_1(\alpha,0)-V_1(0,\alpha)\right)}{\partial\alpha} &= 1-\lambda_1\left(\frac{\partial\alpha(1-\alpha)}{\partial\alpha}F(p^{m_1})+\alpha(1-\alpha)\frac{\partial p^{m_1}}{\partial\alpha}\right)\\
&= 1-\lambda_1\left((1-2\alpha)F(p^{m_1})+\alpha(1-\alpha)(\alpha-\frac{1}{2})\frac{\lambda_1}{4\delta_1}\right)\\
&\geq 1-\lambda_1\left(1-2\alpha+\alpha(1-\alpha)(\alpha-\frac{1}{2})\frac{\lambda_1}{4\delta_1}\right)\\
&\geq 1-\lambda_1(1-2\alpha)\geq 0.
\end{aligned}
$$

In case $\alpha \leq 1/2$, we have that $\alpha(1-\alpha)(\alpha-\frac{1}{2})\frac{\lambda_1}{4\delta_1} \leq 0$. Additionally, we have used the assumption that $\lambda_1 \in [0,1]$. Therefore, $\lambda \in [0,1]$ is a sufficient condition for $\frac{\partial \Phi_1}{\partial \alpha} \geq 0$ in case $\alpha \leq 1/2$. On the other hand, if $\alpha \geq 1/2$, we have

$$
\begin{aligned}
\frac{\partial\left(V_1(\alpha,0)-V_1(0,\alpha)\right)}{\partial\alpha} &= 1-\lambda_1\left(\frac{\partial\alpha(1-\alpha)}{\partial\alpha}F(p^{m_1})+\alpha(1-\alpha)\frac{\partial p^{m_1}}{\partial\alpha}\right)\\
&\geq 1-\lambda_1\left(\alpha(1-\alpha)(\alpha-\frac{1}{2})\frac{\lambda_1}{4\delta_1}\right)\\
&\geq 1-\lambda\frac{\lambda_1}{4\delta_1}\geq 0,
\end{aligned}
$$

which holds if $\delta \geq 1/4$ and $\lambda_1 \leq 1$. Therefore, the two conditions above are sufficient for

$$
\frac{\partial \Phi_1}{\partial \alpha} \geq 0.
$$

A.3.7 Result 6

Define $\widehat{p_1}(\alpha,0) \equiv p^p$, $\widehat{p_1}^g(\alpha,0) \equiv p^g$, and $\delta_1(p-\overline{p_1})^2 \equiv L(p|\overline{p_1})$. Suppose the result does not hold and $p^p \geq p^g$. For optimality of p^g in the external election problem, we have

$$
\begin{aligned}
\varphi^p(p^g)+\varphi^p(p^g)\varphi^g(p^g)-L(p^g|\overline{p}) &\geq \varphi^p(p^p)+\varphi^p(p^p)\varphi^g(p^p)-L(p^p|\overline{p})\\
L(p^g|\overline{p})-L(p^p|\overline{p}) &\leq \varphi^p(p^g)-\varphi^p(p^p)+\varphi^p(p^g)\varphi^g(p^g)-\varphi^p(p^p)\varphi^g(p^p)
\end{aligned}
\tag{26}
$$

On the other hand, from the optimality of p^p in the problem $\mathbb{P}_1$, we have that

$$
\begin{aligned}
\varphi^p(p^p) - L(p^p|\overline{p}) &\geq \varphi^p(p^g) - L(p^g|\overline{p}) \\
L(p^g|\overline{p}) - L(p^p|\overline{p}) &\geq \varphi^p(p^g) - \varphi^p(p^p)
\end{aligned}
\tag{27}
$$

From equations (27) and (26) we get

$$
\varphi^p(p^g) - \varphi^p(p^p) + \varphi^p(p^g)\varphi^g(p^g) - \varphi^p(p^p)\varphi^g(p^p) \geq L(p^g|\overline{p}) - L(p^p|\overline{p}) \geq \varphi^p(p^g) - \varphi^p(p^p),
$$

which implies

$$
\begin{aligned}
\varphi^p(p^g) - \varphi^p(p^p) + \varphi^p(p^g)\varphi^g(p^g) - \varphi^p(p^p)\varphi^g(p^p) &\geq \varphi^p(p^g) - \varphi^p(p^p) \\
\varphi^p(p^g)\varphi^g(p^g) - \varphi^p(p^p)\varphi^g(p^p) &\geq 0.
\end{aligned}
\tag{28}
$$

However, having assumed that the general election platform is more extreme than the primary one, we have that $\varphi^p(p^p) > \varphi^p(p^g)$ and $\varphi^g(p^p) > \varphi^g(p^g)$. Then, it should hold that $\varphi^p(p^g)\varphi^g(p^g) - \varphi^p(p^p)\varphi^g(p^p) < 0$, which contradicts the inequality (28).

A.3.8 Result 7

Using the implicit function theorem, we have

$$
\begin{aligned}
\Phi &= \widehat{p_1}(\alpha_0(\alpha)) - \widehat{p_1}^g(\alpha) \\
\frac{d\alpha_0(\alpha)}{d\alpha} &= -\frac{\partial\Phi/\partial\alpha_0}{\partial\Phi/\partial\alpha} = \frac{\partial\widehat{p_1}/\partial\alpha_0}{\partial\widehat{p_1}^g/\partial\alpha}.
\end{aligned}
$$

We know that $\frac{\partial\widehat{p_1}}{\partial\alpha_0} < 0$. To get $\frac{\partial\widehat{p_1}}{\partial\alpha_0}$ we apply the theorem of the implicit function in the FOCs of the problem with general elections

$$
\Phi^{ex} = 1/2(1-\alpha)(1-\alpha\lambda_1)(1+F^{ex}) + (\alpha + (1-\alpha)(1-\alpha\lambda_1)F)1/2f^{ex} - 2\delta_1(p_1 - \overline{p_1})
$$

$$\frac{\partial \Phi^{ex}}{\partial p_1} = (1-\alpha)(1-\alpha\lambda_1) - 2\delta < 0$$

$$\frac{\partial \Phi^{ex}}{\partial \alpha} = (2\alpha\lambda_1 - 1 - \lambda)(1/2 + F^{ex}) + 1 < 0.$$

Therefore

$$\frac{d\widehat{p_1}^g}{d\alpha} < 0,$$

which implies

$$\frac{d\alpha_0}{d\alpha} > 0.$$

Finally, to prove that if
$\overline{p_1} = -\overline{p_2} = -1/2$ y $\delta_1 = \delta_2 = 2$, then,

$$\alpha_0(\alpha) \approx 0.870595309830104\alpha - 0.437152391546162 \ \text{ if } \lambda_1 = 0$$
$$\alpha_0(\alpha) \approx 0.414021675906255\alpha + 0.0236800496208541 \ \text{ if } \lambda_1 = 1$$
$$\alpha_0(\alpha) \approx 0.337739434101531\alpha + 0.190983005625053 \ \text{ if } \lambda_1 = 2,$$

we do a first-order Taylor approximation around α_0. In the case where $\lambda_1 = 0$, we have

$$\alpha_0(\alpha) = \frac{55.0\alpha^2 + 114.0\alpha - 169.0}{3.0\alpha^2 + 58.0\alpha + 195.0}$$

Applying a Taylor expansion around $\alpha = 0.5$, we arrive at

$$0.870\alpha - 0.437$$

When $\lambda_1 = 1$, we have

$$\alpha_0(\alpha) = \frac{3\alpha^3 + 23\alpha^2 - 87\alpha + 2.0\sqrt{-(3\alpha+5)(\alpha^2+6\alpha-39)(21\alpha^3+57\alpha^2-105\alpha+91)} - 195}{3\alpha^3 + 23\alpha^2 - 87\alpha - 195}$$

$$\alpha_0(\alpha) \approx 0.414\alpha + 0.023$$

When $\lambda_1 = 2$, we have

$$\alpha_0(\alpha) = \frac{1/4\left(18\alpha^3 + 147\alpha^2 - 348\alpha + 1004\sqrt{-0.08\alpha^5 - 0.004\alpha^4 + \alpha^3 - 0.08\alpha^2 - 0.8\alpha + 0.6} - 585\right)}{6\alpha^3 + 49\alpha^2 - 116\alpha - 195}$$

$$\alpha_0(\alpha) \approx 0.337\alpha + 0.19.$$

A.3.9 Proposition 4 4

We use the implicit function theorem to do comparative statics and obtain $\frac{dp_1(\alpha,0)}{d\alpha}$ and $\frac{dp_1(0,\alpha)}{d\alpha}$. The theorem states that[24]

$$\frac{dp_1}{d\alpha} = -\frac{\frac{\partial \Phi}{\partial \alpha}}{\frac{\partial \Phi}{\partial p_1}}.$$

Note that evaluated at the optimum

[24]This corresponds to the implicit function theorem for the case of $f : \mathbb{R}^N \to \mathbb{R}$.

$$\Phi = -1 \cdot \left(G' \cdot (-2A(\alpha) + 2p_1)\right) - 2\delta_1(p_1 - \overline{p_1}) = 0.$$

Partially differentiating Φ with respect to p_1

$$\frac{\partial \Phi}{\partial p_1} = -1 \cdot \left(G'' \cdot (-2A + 2p_1)^2 + 2G'\right) - 2\delta_1 < 0.$$

Partially differentiating Φ with respect to α

$$\frac{\partial \Phi}{\partial \alpha} = -1 \cdot \left(G'' \cdot \left(2A'(\alpha) \cdot (p_2 - p_1) \cdot (-2A(\alpha) + p_1)\right) + G'(-2A'(\alpha))\right).$$

Let's analyze the sign of this expression. For $\frac{\partial \Phi}{\partial \alpha}$ to be negative, we need

$$\begin{aligned}
&G'' \cdot \left(2A'(\alpha) \cdot (p_2 - p_1) \cdot (-2A(\alpha) + p_1)\right) + G'(-2A'(\alpha)) > 0 \\
&\frac{G''}{G'} \cdot (p_2 - p_1) \cdot 2(p_1 - A(\alpha)) < 1 \\
&\frac{G''}{G'} > \frac{1}{(p_2 - p_1) \cdot 2(p_1 - A(\alpha))},
\end{aligned}$$

which is true whenever $G'' > 0$, since $p_2 - p_1 > 0$ and $p_1 - A(\alpha) < 0$. For the previous inequalities we use the fact that $A' < 0$ since

$$\begin{aligned}
A(\alpha) &= F^{-1}\left(\frac{1 - 2\alpha}{2(1 - \alpha)}\right) \\
\frac{dA(\alpha)}{d\alpha} &= \frac{1}{f\left(F^{-1}\left(\frac{1-2\alpha}{2(1-\alpha)}\right)\right)} \cdot \left(\frac{-1}{(1 - \alpha)^2}\right) < 0.
\end{aligned}$$

Consequently

$$\frac{dp_1(\alpha, 0)}{d\alpha} < 0 \text{ siempre que } G'' > 0.$$

Next, we see what happens to the candidate's platform when he does not win the Cacique's votes.

$$\frac{\partial \Phi}{\partial p_1} = -1 \cdot \left(G'' \cdot (-2A + 2p_1)^2 + 2G' \right) - 2\delta_1 < 0$$

Differentiating with respect to α

$$\frac{\partial \Phi}{\partial \alpha} = -1 \cdot \left(G'' \cdot \left(2A'(\alpha) \cdot (p_2 - p_1) \cdot (-2A(\alpha) + p_1) \right) + G'(-2A'(\alpha)) \right)$$

Let's analyze the sign of this expression.

$$\begin{aligned}
& G'' \cdot \left(2A'(\alpha) \cdot (p_2 - p_1) \cdot (-2A(\alpha) + p_1) \right) + G'(-2A'(\alpha)) > 0 \\
& \frac{G''}{G'} \cdot (p_2 - p_1) \cdot 2(p_1 - A(\alpha)) > 1 \\
& \frac{G''}{G'} < \frac{1}{(p_2 - p_1) \cdot 2(p_1 - A(\alpha))}
\end{aligned}$$

In this case, unlike the previous case in which Candidate 1 won the votes, now $A'(\alpha) > 0$. Consequently

$$\frac{dp_1(0, \alpha)}{d\alpha} < 0$$

if

$$\frac{G''}{G'} < \frac{1}{2(p_2 - p_1)(p_1 - A)},$$

and

$$\frac{-\delta - 2G'}{G'} < \frac{G''}{G'}.$$